

PROJECTIVE GEOMETRY IN LOGICAL SPACE:
RETHINKING TRACTARIAN THOUGHTS¹

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Customary interpretations state that Tractarian thoughts are pictures, and, *a fortiori*, facts. I argue that important difficulties are unavoidable if we assume this standard view, and I propose a reading of the concept taking advantage of an analogy that Wittgenstein introduces, namely, the analogy between thoughts and projective geometry. I claim that thoughts should be understood neither as pictures nor as facts, but as acts of geometric projection in logical space. The interpretation I propose thus removes the root of the identified difficulties. Moreover, it allows important clarification concerning some central aspects of the Tractarian theory of representation, and it yields a unifying elucidation regarding Wittgenstein's remarks on the solipsistic thesis.

Keywords: Wittgenstein, *Tractatus Logico-Philosophicus*, thoughts, projective geometry, logical space.

1. INTRODUCTION

The concept of thought is introduced in the *Tractatus* as an intermediate step between the exposition of the general theory of representation (2.1-2.225)² and its crucial application to linguistic propositions (starting in 3.1). The concept appears only in 13 propositions in the *Tractatus*³, and Wittgenstein's laconism on this matter makes it rather difficult to come up with an adequate understanding of the notion. In this article I attempt such a task from a critical point of view. In sections 2 and 3, I construe two general standard interpretations that try to remain faithful to the text, and I show that both of them are ultimately untenable. I argue that the problems with these customary interpretations result from the view that thoughts are pictures and, hence, facts. In section 4, I introduce a reading of the concept of thought that takes seriously, and elaborates on, the analogy with the method of projective geometry that Wittgenstein makes use of: I propose that Tractarian thoughts can be fruitfully understood as acts of geometric projection in logical space, performed by the transcendental subject. The interpretation that I introduce removes the root of the problems I identify in the customary readings, for in my proposal thoughts are neither pictures nor facts. Besides, my construal has further advantages that I present and explain in section 5: it allows important clarification with respect

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² All textual references to the *Tractatus* are to Wittgenstein (2002).

³ Namely, in propositions 3, 3.01, 3.02, 3.04, 3.05, 3.1, 3.12, 3.2, 3.5, 4, 4.002, 4.112, and 6.21.

to some essential aspects of the general theory of representation, and it also sheds light on the difficult issue of the solipsistic thesis. In fact, the doctrine of thoughts as I present it and Wittgenstein's remarks about the solipsistic thesis are essentially connected and unified. In section 6, I provide a conclusive summary and I show that my interpretation must drop at least one Tractarian proposition (3.2), for that proposition is undeniably committed to the view that thoughts are facts. However, any interpretation that intends to avoid the problems I identify in sections 2 and 3 must do something similar—that is, I claim that Wittgenstein's literal doctrine of thoughts is essentially problematic.

2. THE NATURAL INTERPRETATION

In order to formulate what I call the natural interpretation of the concept of thought it is useful to quickly review the essentials of the Tractarian theory of representation. To begin, we have that a picture can depict a fact because it has a structure that is similar to the structure of the depicted fact. That is, a picture is able to represent a fact insofar as the elements in the former are somehow similarly arranged as the elements in the latter. Wittgenstein dubs this connection between picture and depicted fact the *pictorial form* (2.15-2.151). In turn, the pictorial form grounds the *pictorial relationship*, i.e., that each element in the picture stands for an element in the depicted fact (2.13, 2.131, 2.1513, 2.1514). If representation necessarily requires pictorial form and pictorial relationship, then all pictures are facts (2.141), for facts are constituted by elements configured in a certain way (2.01).

Pictures can take all sorts of pictorial forms. In some pictures it is the spatial organization of their elements that mirrors the structure of the depicted facts, in other pictures it is the temporal organization what plays that role, and we can imagine innumerable ways in which the pictorial form can get instantiated. Now, notwithstanding the concrete instantiation of the pictorial form, for any picture to be a picture at all, it must have a structure in common with the depicted fact. This requirement is expressed in Wittgenstein's concept of *logical form*: "What any picture, of whatever form, must have in common with reality, in order to be able to depict it—correctly or incorrectly—in any way at all, is logical form, i.e., the form of reality" (2.18). In other words, there can be pictorial form only if there is logical form. Therefore, Wittgenstein states that "every picture is *at the same time* a logical one" (2.182), where a *logical picture* is defined as "a picture *whose pictorial form is logical form*" (2.181, my emphasis).

The concept of *thought* makes its first appearance in proposition 3, where we read that "a logical picture of facts is a thought". That thoughts are defined as logical pictures seems to obviously entail that thoughts are pictures, and if thoughts are pictures, they must be facts (2.141). Finally, considering propositions 2.181 and 3, thoughts are naturally (and often) understood as pictures whose pictorial form corresponds to nothing but their logical form. A succinct and precise formulation of this view is presented, for example, by Anthony Kenny: "every picture is, of course, a logical picture; but thoughts are logical pictures *par excellence* since logical structure is the *whole* of their pictorial form" (Kenny 2006, 68).

In sum, in a natural and literal reading of Tractarian thoughts we have two main aspects. First, they are pictures, and hence facts, whose elements are arranged in a way that is isomorphic to the way that the elements in the depicted fact are arranged — this reading is supported by propositions 2.141, 2.15, 2.151, 2.181, 3 and 3.2. Second, since thoughts are logical pictures, the similarity of structure involved is only logical, so a thought is neither a spatial-figure, nor a temporal-figure, nor a color-figure, etc. — this view is supported by 2.18, 2.181, 2.182, and 3.

The view that thoughts are facts composed by elements arranged in a certain way is explicitly endorsed by Wittgenstein in the context of an often quoted 1919 epistolary discussion with Bertrand Russell. When Russell, after reading a draft of the *Tractatus*, asked Wittgenstein what the elements that compose a thought are, Wittgenstein answered:

But a *Gedanke* is a *Tatsache*. I don't know what the constituents of a thought are but I know that it must have such constituents which correspond to the words of language [...]. The kind of correlation of the constituents of thought and of the pictured fact is irrelevant. It would be a matter of psychology to find out. (Wittgenstein 1961, 129)

Furthermore, when Russell goes on to suggest that those elements may be words, Wittgenstein replies that “does a *Gedanke* consist of words? No! But of psychical constituents that have the same sort of relation to reality as words. What those constituents are, I don't know” (*ibid*, 130).

An interpretation in which thoughts are logical pictures, i.e., facts whose isomorphy with the depicted fact is purely logical, is problematic. The discussion between Russell and Wittgenstein is useful to illustrate the difficulty. If thoughts are facts composed of certain psychical constituents—or of any kind of concrete constituents—the thought certainly has an instantiated pictorial form that makes it a picture of the corresponding fact. The constituent psychic elements may be arranged either in a spatial way, or in a temporal way, etc., but they are surely arranged in a concrete manner. Therefore, the pictorial form of the thought cannot reduce to, or be identified with, its logical form.

The reference to *psychological* elements may suggest an interpretation of the *Tractatus* in which thoughts do not really have a concrete structure—thus avoiding the problem. If we conceive the psychical elements as some kind of abstract immaterial entities, their arrangement in a thought may not be concrete, but abstract and merely logical. In simple words, we may conceive that there are abstract facts which are purely logical, and that thoughts correspond to that category of facts. So conceived, the isomorphy between a thought and the corresponding depicted fact may restrain to logical form.

There are two reasons why I think such an interpretation is untenable. First, Wittgenstein states in his discussion with Russell that the task of finding out what the elements of a thought are is a matter of psychology, a natural science about concrete facts. Indeed, proposition 4.1121 states that “psychology is no more closely related to philosophy than any other natural science”.

Secondly, 3.2 states that there is an isomorphic relation between a proposition and the corresponding thought: “In a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the objects of the thought”. Furthermore, in his reply

to Russell Wittgenstein states that the constituents of a thought have the same sort of relation to reality as words. This isomorphy between thoughts and propositions presupposes that thoughts are facts with a determinate structure (cf. 3 and 2.141), and it is the factual character of thoughts that grounds the problem I am calling attention to. Proposition 2.01 defines a fact (*Sachverhalt*) as a combination of objects, and this definition certainly involves that the combination is arranged in a concrete manner. We can illustrate this point recalling that a central tenet in the *Tractatus* is that the truth value of a picture cannot be known *a priori*, all pictures are empirical (2.222-2.225). Now, if a thought is a fact (3, 3.2), it can certainly be depicted, and we can only get to know the truth value of the picture that depicts it by means of empirical enquiry, and how could we get to know, by means of empirical examination, that a certain fact—the depicted thought—is the case or not if it is not concrete? We can only get to know that a picture is true—i.e. that the represented fact is the case—by examining the world, and in this sense *all* facts are concrete⁴.

The upshot is then that the problem with the natural interpretation of thoughts is that if a logical picture is a picture whose pictorial form is solely logical, that picture cannot be a fact. If it were a fact, its elements would be arranged in a specific form, so it would have a pictorial form that exceeds its logical form. But if a logical picture cannot be a fact, then it cannot be a picture. In short, an interpretation in which thoughts are facts and logical pictures (pictures whose pictorial form is solely logical) is inconsistent.

3. AN ALTERNATIVE INTERPRETATION

One way to avoid this problematic result is to drop the second aspect in the natural interpretation, namely, the view that in thoughts the pictorial form reduces to the logical form. But then we must interpret the propositions that seem to support such a view in a suitable way. Let us try to do that. Proposition 2.182 states that “every picture is *at the same time* a logical one. (On the other hand, not every picture is, for example, a spatial one)”. In order to avoid the identity between pictorial and logical form in the case of logical pictures, we must understand this sentence as only asserting that logical form is a necessary condition for pictorial form. That is, every picture contains a logical picture in the sense that the pictorial form presupposes logical form, but we should not infer from this that there can be pictures whose pictorial form is purely logical. Accordingly, proposition 3, which states that “a logical picture of facts is a thought” should be understood as stating that a thought, *just as any other picture*, is a

⁴ Actually, in a literal reading of the *Tractatus* propositions are indeed pictures of thoughts (see section 3 below). In 3.1 we read “In a proposition a thought finds an expression that can be perceived by the senses”, and as we saw, 3.2 states that there is an isomorphy between thoughts and propositions, so the latter are pictures of the former. We find an interesting issue here. As a pictures *of a corresponding thought*, propositions are always true. A proposition represents a certain possible fact in reality (*Wirklichkeit*) *via* the depiction of a thought which is isomorphic with the possible fact. But if the mediating thought were a fact which is not the case, the proposition would not be a depiction of the possible fact, for the mediating role of the thought would not be fulfilled. This cannot mean, though, that as pictures of a thought propositions are true *a priori*. The truth of a proposition as a picture of a thought is still a contingent matter of experience, which reinforces my point that thoughts are concrete facts.

logical picture in the sense that there is a logical form that makes it possible for the thought to have a pictorial form.

In this reading, the role that thoughts play in the theory of representation cannot be that they are pictures whose pictorial form is purely logical, but that they constitute a kind of pictures underlying all ordinary pictures. Now, this special role can be understood in two different ways. In a *mentalist* view, thoughts correspond to the psychological facts that establish the representative character of the propositional sign (the picture that expresses the thought). Without a thought supporting and underlying them, signs (facts with a certain structure) cannot become symbols (pictures with pictorial form). That is, pictures can represent only inasmuch as they contain (or, perhaps more precisely, inasmuch as they depict) a thought. According to this reading, thoughts are the primordial pictures of facts, and all ordinary perceivable pictures become representative insofar as they represent, or contain, a thought. An interpretation of this type is endorsed, for example, by Norman Malcolm (1997, 32-33) and by Severin Schroeder:

It is thinking that, by expressing itself in words, enables those words to depict the world. Thoughts, unlike conventional noises or signs, are intrinsically models of the world. *Linguistic signs become models of the world only by courtesy of the thoughts that express themselves in them.* For this reason, Wittgenstein, after giving a general exposition of the picture theory, applies it first not to language, but to thoughts (TLP 3-3.05). 'A logical picture of facts', Wittgenstein writes, 'is a thought' (TLP 3); *every picture is also a logical picture* (TLP 2.182): *every picture is or expresses a thought. Thought is, as it were, the lowest common denominator of all different kinds of pictures* (paintings, construction drawing, verbal description, musical score or wordless pondering). Through signs a thought becomes public: the picture can now be perceived by the senses (TLP 3.1). What the English sentence 'It's raining' and the French sentence 'Il pleut' have in common is that they can be used to express the same thought, by means of different linguistic conventions. Like these statements, the thought expressed is also a picture of a situation in which it is raining, but it is independent of any linguistic stipulations. It is depiction pure and simple. (Schroeder 2006, 61-62, emphasis added)

The second way in which the role of thoughts is interpreted is *anti-mentalist*. In this outlook, thoughts, and psychological facts in general, do not play an essential role in turning signs into symbols. It is the logical form that thoughts and propositions share with the represented fact what makes them potential pictures of that fact, and this a matter of logic (the structure of logical space), not a matter of psychology. From an anti-mentalist standpoint, thoughts are not primordial pictures, but pictures like any other. However, there must be a correlation between a certain propositional sign and a thought that actualizes the potential pictorial power of the sign, and *this* is a matter of psychology—the main point in the anti-mentalist stance is that the representational power of pictures is not grounded on psychological aspects. This view is endorsed, for example, by Mounce:

But, it might be said, it is surely *we* who correlate the elements in the proposition with the world, and, therefore, it is *we* who give sense to the proposition. The answer is that 'correlation' is ambiguous. What is obviously true is that a mark does not correlate itself with the world; somebody has to do something; *some psychological activity is necessary if the correlation is to occur.* (What

the psychological process might be is [...] entirely irrelevant). Now, if one wishes, one can call this 'correlation'. But the point is that if by 'correlation' one means a logical connection then it is not a psychological activity. In short, the psychological activity, though necessary if the correlation is to occur, does not in itself bring about the logical connection between name and meaning. This is brought about by the logical structure into which the mark enters. (Mounce 1989, 33)

Notice that under this view psychological facts still play a role in the foundations of pictorial representation, though a weaker one than in the mentalist approach. Albeit the logical bases for representation do not depend on psychology, and therefore do not depend on thoughts as an expression of psychological activity, thoughts are still crucial in actualizing the representational power of a potential picture. Notice also that although the details of the psychological process are philosophically irrelevant, it must certainly be a process in which thoughts are isomorphic with the facts they represent.

There are two main problems with this alternative interpretive avenue (regardless of the mentalism/anti-mentalism issue), one exegetical and one philosophical. First, we have proposition 2.181, which we saw states that "a picture *whose pictorial form is logical form* is called a logical picture" (my emphasis). The only way we can make sense of this assertion, while avoiding the problem mentioned in the previous section regarding pictures whose pictorial form is purely logical, is by taking it as stating only that a picture is a logical picture if its pictorial form is made possible by logical form. I think this is a rather forced understanding of the sentence. The German formulation reads "*Ist die Form der Abbildung die logische Form, so heißt das Bild das logische Bild*" (my emphasis). This sentence clearly suggests that there are some pictures in which the pictorial form *is* the logical form, rather than that the pictorial form of all pictures is made possible by the logical form. Besides, the adapted interpretation makes the proposition almost trivial: its only content would be to stipulate that when we say that a picture is a 'logical picture' we are simply emphasizing that the pictorial form is grounded on logical form⁵.

Proposition 3 is even less adaptable to this alternative interpretation. The German formulation reads: "Das logische Bild der Tatsachen ist der Gedanke". If Wittgenstein's intention were to underscore that, just as in any other picture, the pictorial form of a thought makes possible its pictorial form, he would have probably inverted the roles of subject and predicate in this sentence, but the choice of "Das logische Bild der Tatsachen" as the sentence subject clearly suggests that he is identifying thoughts with a special kind of pictures: all and only those pictures in which the pictorial form is the logical form.

⁵ Max Black endorses a view of this type: "A picture counts as a 'logical picture' (soon to be identified with a proposition) when the [pictorial form] is the logical form of representation, i.e., when the structure of the picture-fact shows that the co-ordinated objects *can be* structured in exactly the same manner (i.e. by concatenation) and when the picture means ('says') that the objects are thus combined. Since every picture can be regarded as a fact, every picture satisfies the essential conditions for being a logical picture" (Black 1964, 89-90).

Anyhow, we could just take the stance that propositions 2.181 and 3 are, literally understood, essentially problematic, so that if we cannot suitably interpret them, we may better drop them. Actually, I will argue below that something like this must be done anyway.

However, and notwithstanding these exegetical considerations, there is second problem—more philosophical and serious—with the alternative interpretation. To state, as Wittgenstein does, that thoughts are facts made up of psychological constituents that pictorially represent other possible facts is an exercise in *a priori* armchair neuroscience. The pictorial theory of representation is not a description of how (certain kinds of) facts are disposed in the world, but a transcendental study on the conditions of possibility of the representation of facts. It could be plainly false, from a neurophysiological point of view, that the material basis of thoughts are given by psychic elements that stand in an isomorphic relationship with objects in the world (given the gargantuan complexity of brain processes, this is actually a naturally expected result) and yet the Tractarian theory of representation still be adequate from a philosophical point of view. Actually, one may expect that the philosophical value of the pictorial theory of representation does not depend at all on how neural-psychological facts are disposed. Erik Stenius, for example, endorses a reading of Tractarian thoughts of this type, in which the problematic commitment to armchair neuroscience is crystal clear:

Pictures of facts concerning red flowers or green leaves need not have the colors 'red' and 'green' as elements [...] there must only be a structural similarity between a picture and its prototype. We need only take this fact into consideration to give an account of 'thinking' as a sort of depicting which proceeds as easily on a completely physiological as on any other basis for psychology. From a physiological standpoint we might venture a description of the following kind: Not only thinking, but also perceiving, consists of the formation of an isomorphic 'neural' picture of reality, the elements of which are some entities belonging to the neural system as an articulate field. (Stenius 1996, 113)

I think that making the pictorial theory of representation to rely (either in the strong mentalist sense or in the weak anti-mentalist sense) on psychological facts—or on any kind of empirical facts—is an untenable position. These critical remarks become even more stringent if one considers the following propositions:

4.1121 Psychology is no more closely related to philosophy than any other natural science.

Theory of knowledge is the philosophy of psychology.

Does not my study of sign-language correspond to the study of thought-processes, which philosophers used to consider so essential to the philosophy of logic? Only in most cases they got entangled in unessential psychological investigations, and with my method too there is an analogous risk.

4.1122 Darwin's theory has no more to do with philosophy *than any other hypothesis in natural science*. [My emphasis]

We should definitively avoid the risk that Wittgenstein mentions. The pictorial theory of representation cannot assume a certain description of concrete facts, either mental or of any type. So if the Tractarian concept of thought is to play a role in the pictorial theory, it cannot

refer to a specific kind of empirical facts. We should then explore the possibility to formulate an interpretation in which thoughts are not facts, but such an interpretation opens the challenge that they cannot be pictures.

4. THOUGHTS AS ACTS OF PROJECTION

So far I have not made reference to an analogy that Wittgenstein offers to explain the concept of thought. I will now argue that this analogy is the key to a cogent interpretation. In propositions 3.1-3.13 Wittgenstein brings up the concept of *projection* in order to present the connection between thoughts and propositions. *Projection* is a geometrical term. *Projective geometry* was developed by the great mathematician Felix Klein in the second half of the XIX century, and engineering methods were developed from it, which were usually taught under the name of *descriptive geometry*. As Hamilton (2001) reports, during his engineering formation in Berlin Wittgenstein attended several courses on projective-descriptive geometry, and it is most likely that his studies on these mathematical methods were very important, in historical-genetic terms, in the formulation of the pictorial theory of representation. Unlike Wittgenstein's acquaintance with the method of representation of car accidents in Parisian courts, his academic training in projective methods is not usually mentioned as a conceptual seed for the pictorial theory of representation. These historical-genetic remarks motivate the insight I will now develop. I intend to make the point that exploiting the projective analogy allows us to formulate an adequate and fruitful interpretation of Tractarian thoughts.

The analogy has not been completely overlooked in the literature, but it has not received the attention it deserves and it has not been developed in an interpretively fertile way. To cite a few representative examples, Anscombe confines the analogy to a footnote, where she provides an explanation that is not quite correct⁶. Black vaguely refers to the geometrical origin of the notion of projection, but he does not assign it any clarifying or interpretive role⁷. Russell, in his introduction to the *Tractatus*, refers to the analogy, but without connecting it with the notion of thought⁸. Kenny uses the analogy, without explaining it in detail, to argue that thoughts are a sort of intermediate pictures linking propositions and facts⁹. Fogelin (1995, 27-

⁶ "Wittgenstein's use of 'projection' is a metaphorical extension of the mathematical use, which may be explained thus: 'The drawing of straight lines through every point of a given figure, so as to produce a new figure each point of which corresponds to a point in the original figure'. The new figure is also said to be a projection of the original one, which is projected *into* it". Anscombe (1965, 69).

⁷ "The idea of 'projection' is no doubt suggested by the 'projection' of a geometrical figure on to a plane (e.g. in descriptive drawing). It can serve as a reminder of the necessity of some rule or 'law of projection' (cf. 4.0141) and of the 'distortion' resulting—the 'accidents' of the resulting representation. For many purposes, 'project' (*projizieren*) can be taken as synonymous with 'present' (*darstellen*) or 'depict' (*abbilden*)". Black (1964, 99).

⁸ "He compares linguistic expression to projection in geometry. A geometrical figure may be projected in many ways: each of these corresponds to a different language, but the projective properties of the original figure remain unchanged whichever of these ways may be adopted. These projective properties correspond to that which in his theory the proposition and the fact must have in common, if the proposition is to assert the fact". (Wittgenstein 2002, ix-x).

⁹ "Thought appears in the *Tractatus* mainly as the link between propositions and states of affairs. To illustrate this, Wittgenstein often uses the metaphor of geometric projection. The propositional sign (the spoken or

29) and Mounce (1989, 29-34) do not pay much attention to the concept of projection, and they do not assign a geometrical meaning to it¹⁰.

The basic idea in projective geometry is simple. Take two planes Γ and H , and a figure Φ on Γ . P is a point outside both planes, from which we draw straight lines that intersect each point of figure Φ , and that continue to intersect points in plane H ¹¹. The figure ψ formed by the latter points constitute the projection of Φ on H . Notice that the points that constitute the projection-figure ψ are already given in space. The process, in a word, consists in establishing a bijective correlation between two sets of points: the set of points that constitute figure Φ , and the set of points that constitute its projection ψ .

The mathematical usefulness of this method relies on the fact that, notwithstanding that the projected figure and its projection are different figures, some geometric properties of the former are preserved in the latter – these are the projective properties. That the projective relation between the figures is possible, and that they share certain geometric properties, is determined by the structure of the embedding space¹². For example, if the projected figure Φ is a conic section (a circle, an ellipse, a parabola or a hyperbola), its projection in H is also a conic section. The following figure illustrates the method: Φ is a circle on the plane Γ and its projection on the plane H is a parabola. The projective lines and the projective properties are determined by the structure of the embedding space that contains P , Γ and H ¹³.

written sentence), when it is used, is a projection of a possible state of affairs; the method of projection is to think of, i.e., to form a logical picture of, the possible state of affairs which is the sense of the proposition (TLP 3.5, 3.11-3.12). It is the thought which makes the sign into a symbol, which makes the propositional sign into a symbol; so much so that sometimes Wittgenstein speaks of a proposition as actually being a thought (TLP 3.5-4) and not just, as seems more natural, as being the expression of a thought (3.2). The projection lines as it were, run from the sentence to the state of affairs *via* the thought in the mind". (Kenny 2006, 48).

¹⁰ Perhaps they think that it is too obvious that projection is a geometric term, which would be a confirmation that they do not assign much importance to the analogy.

¹¹ Anscombe's explanation of the method of projection is not quite correct inasmuch as she does not refer to the point P , which plays an essential role in the mathematical method, and also in my interpretation of thoughts as geometrical projections in logical space.

¹² There are three main levels of geometric structure. Topological structure describes the way in which the points in a manifold are connected. Affine structure establishes a distinction between straight and curved lines (more precisely, it distinguishes geodesics from other lines). Metric structure establishes the distances between points in the manifold. Euclidean and non-Euclidean geometries (hyperbolic and spherical), differ in terms of their affine and metric structure. The specific projective properties which are conserved in projections can discern between spaces with different metrics: what are the projective properties depends on the metric. This explains my statement that the possibility of the projective lines and the projective properties is determined by the structure of the embedding space. For a simple explanation of the levels of geometric structure, see (Maudlin 2012, Ch. 1)

¹³ For a simple account of projective geometry, see Torretti (2016). My explanation strongly relies on this source.

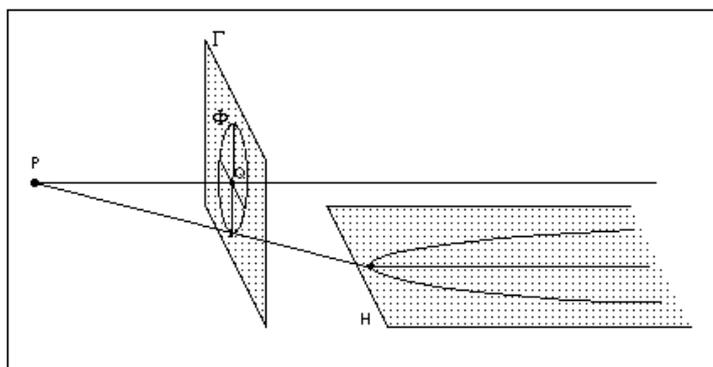


Fig. 1. Projective geometry. Reprinted from Torretti (2016)

The view I would like to propose here is that Tractarian thoughts are not to be understood as facts nor as figures, but as the activity of establishing a logical-representational connection between facts in logical space, according to this method of geometric projection. In short, Tractarian thoughts can be understood as the application of this method of geometric projection within logical space.

By interpreting the concept of thought as the activity of projecting in logical space we remove the root of the problems that affect the interpretations reviewed in sections 2 and 3. If thoughts are understood as a (transcendental, not psychological) *activity* that result in pictures, they are no longer facts. Thus, we avoid the problem of pictures whose pictorial form is strictly logical, and also the problem of doing armchair neuroscience. On the other hand, the specifics of the analogy with geometric projection allow a conception of thoughts which is quite coherent with the Tractarian theory of representation.

Logical space is one of the most complex concepts in the *Tractatus*, but, in a nutshell, it can be defined as the structure of reality (*Wirklichkeit*). Let us recall that Tractarian objects (*Gegenstände*) necessarily occur in elementary states of affairs (*Sachverhalte*)¹⁴. The internal properties of an object determine all the states of affairs in which that object can occur, but which of those states of affairs the object effectively appears in is a contingent issue. Now, the internal properties of the totality of objects determine logical space, that is, the underlying structure of reality as the totality of possible states of affairs.

I claim that a cogent interpretation is that thoughts are the acts of tracing projective lines from the transcendental subject (point P), passing through objects in a possible state of affairs (the points in figure Φ), and that reach objects in another possible state of affairs (the points in the projected figure ψ on H). The latter atomic fact ψ is the projection, and it plays the role of representing the projected state of affairs (Φ)¹⁵.

¹⁴ Given Wittgenstein's fluctuating use of *Sachverhalt*, *Tatsache* and *Sachlage*, there are several ways to interpret these terms. I will take *Sachverhalt* for *elementary state of affairs*, *Tatsache* for *fact* in general, and *Sachlage* for *situation* or *the existence and non-existence of states of affairs* (2.11). More concretely, assuming that p and q are elementary propositions, then p and q are *Sachverhalte* and they represent *Sachverhalte*. The proposition $p \vee \neg q$ is a *Tatsache*, but not a *Sachverhalt*, and it represents a *Sachlage*, the situation given by the existence of the *Sachverhalt* represented by p and the non-existence of the *Sachverhalt* represented by q .

¹⁵ It is important to underscore that since geometric projection occurs in logical space, which is composed of all possible states of affairs (*Sachverhalte*), then the projected facts and their projection are states of affairs and elementary propositions, respectively. That is, the outcome of projections in logical space are elementary

Notice that, just as in the mathematical case explained above, the objects that constitute the projection are already given in logical space. Thus, there is no duplication or transformation of states of affairs into other states of affairs. Projection in logical space simply establishes a bijective correlation between two sets of objects: the objects that constitute the projected fact (Φ), and the objects that constitute its projection-fact (ψ).

Furthermore, just as in spatial geometry the possibility of the projective lines and the shared projective properties are determined by the structure of the embedding space; the possibility of the projective lines in a thought and the shared properties between the projected fact and its projection (the isomorphy), are determined by the structure of logical space. In other words, the specific arrangement of the constituent objects in both facts involved, and also the projective lines which constitute the logical form that grounds the isomorphy between them, are determined by the form of reality — i.e., by the structure of logical space.

This particular aspect of the analogy between projective geometry and thoughts is highly relevant and fruitful. As we will see below, the structure of logical space and logical form are explicitly identified in the *Tractatus*, so in my interpretation we get a simple illustration that logical form and projection in logical space (i.e., thoughts) are essentially related, without falling in the problems that arise in the interpretations presented in sections 2 and 3.

Since thoughts consist in the activity of applying the method of projection in logical space, we avoid the problematic view that thoughts are facts composed of psychical elements. In my proposal thoughts are not facts at all, the only facts involved are the projected fact and its projection, i.e., the represented state of affairs and the picture that represents it. We thus avoid the problematic issues in the natural interpretation reviewed in section 2.

On the other hand, since the projection occurs in logical space, the subject involved is not a psychological mind, but a transcendental subject. Under my proposal, thoughts are understood as one of the transcendental conditions for representation that configure the metaphysics of the *Tractatus*. That is, geometric projection in logical space is not a matter of empirical facts occurring in space and time, so we can distinguish thoughts in this transcendental sense (as a condition for the possibility of representation) from the *psychological* activity of thinking. In this interpretation we are no longer philosophers turned into neuroscientists.

A way to further explain and clarify the interpretation I propose is by contrast with a view presented by Hacker that is in some sense similar to mine:

Thought constituents must, of course, possess the appropriate mathematical multiplicity to depict the facts. Hence they must correspond to the names in a fully analysed language. That such configurations, in thought or language, *actually* represent (and do not merely contain the possibility of representing (*TLP* 3.13)) is a function of the will, of the metaphysical self [...]. It is a mental act (albeit of a transcendental self, not of the self that is studied by psychology) that injects meaning or significance into signs, whether in thought or in language. One might call this conception 'The Doctrine of the Linguistic Soul', for it is the soul that it is the fountainhead of language and representation. (Hacker 1986, 75)

propositions (or more generally, elementary pictures). Molecular propositions are built out of elementary propositions, and through the operation of truth-functional logical connectives they represent situations (*Sachlagen*).

I agree with the basic tenet of the ‘doctrine of the linguistic soul’, namely, that it is the transcendental self that injects representational power into signs. However, since this doctrine as described by Hacker is still committed to a conception of thoughts as facts, it falls prey to the problem of armchair psychology. Furthermore, this view of the linguistic soul is still unclear. In a later work, Hacker refers critically to it: “one might argue that thoughts-constituents *are* related to reality as words are, i.e., extrinsically. The mechanism of correlation would be by means of the Will, but not the Will as a phenomenon, rather the Will as an aspect of the meta-physical self. This, however, is mere mystery-mongering” (Hacker 1990, 315). I think that the description of thoughts as geometric projection contributes to dispel the purported mystery (I will elaborate on this below), and it removes the root of the identified problems, namely, that thoughts are facts whose constituents are psychical elements that pictorially represent¹⁶.

5. ADVANTAGES OF THIS INTERPRETATION

This reading of Tractarian thoughts allows us to make sense of essential aspects of the pictorial theory of representation. For starters, the notion of pictorial form is quite coherent with my proposal. In projective geometry, the projective properties are features of the projected figure that are conserved in its projection. In the example above, a circle is projected onto a different figure, a parabola, but both are conic sections. More generally, the projection has some structural properties in common with the projected figure. This is quite analogous to the notion of pictorial form. In my rendition of thoughts, pictorial form simply corresponds to the projective properties.

The pictorial relationship is also easily grasped in my interpretation. In logical space, the point in which a projective line intersects the projection is the name of the point in which the same line intersects the projected fact. Now, a single line in logical space cannot count as a projective line. A point has no figure-properties, so it cannot be projected in such a way that its projection preserves some of its properties. This is illustrative of Wittgenstein’s view that names can have a meaning only in the context of a proposition (3.3).

Another central Tractarian tenet is that the fact represented by a figure is a possible fact, i.e., a state of affairs in logical space (2.201-2.203)—this is why we can represent states of affairs which are not the case. Since in my proposal thoughts are projections in logical space, this is a natural result—the projected-represented fact is a possible state of affairs. In turn, that there

¹⁶ When Hacker tackles the subject that thoughts allegedly inject meaning into signs, he states that “further complication and irresolvable tension were added by the account of a thought as a psychological fact” (1990, 314). I agree. However, the irresolvable tension that Hacker has in mind is different than the one I am addressing: “The tension in this account is between the idea that a thought is a representation and thinking a kind of language, on the one hand, and the idea (as Wittgenstein later put it) ‘that thought is the last interpretation’”. That is, Hacker states that it is very difficult to jointly endorse the thesis that thoughts are the primordial and ultimate sources of representational meaning, and the thesis that they are regular pictures composed of psychic elements. I state that the view that thoughts are psychic facts is problematic in and by itself.

are no *a priori* true or false pictures—another basic Tractarian thesis (2.21-2.225)—is also a natural outcome in my interpretation. The truth or the falsity of the projection depends on whether the projected state of affairs is the case or not, and this cannot be known *a priori*.

Wittgenstein also states that propositions are perceivable expressions of thoughts (3.1). We can clearly make sense of this principle in our interpretation: if the projection is a state of affairs that is the case—a fact in the world (*Welt*)—then the thought becomes perceivable and publicly accessible. This is clearly so when the projection in a determinate thought is a proposition: the projective lines go from *P*, they pass through the elements in the projected state of affairs, and reach the names in the proposition (the projection). Actually, this same principle works for all types of pictures, propositions are simply a special kind (a very important kind, of course) of facts in the world that make thoughts perceivable by the senses. We have then that a thought results in a perceivable picture when the projection is an existent fact, and that an underlying thought is what turns a perceivable sign into a symbol with a sense. These two theses can be confirmed by a direct contrast with Wittgenstein's words:

3.1 In a proposition a thought finds an expression that can be perceived by the senses.

3.11 We use the perceptible sign of a proposition (spoken or written, etc.) as a projection of a possible situation.

The method of projection is to think the sense of a proposition.

3.12 I call the sign with which we express a thought a propositional sign. — And a proposition is a propositional sign in its projective relation to the world¹⁷.

3.13 A proposition includes all that the projection includes, but not what is projected.

Therefore, though what is projected is not itself included, its possibility is.

A proposition, therefore, does not actually contain its sense, but does contain the possibility of expressing it.

(‘The content of a proposition’ means the content of a proposition that has sense).

A proposition contains the form, but not the content, of its sense.

Proposition 3.1 states that a thought is expressed in a perceivable way by means of a proposition (or by means of any concrete picture, recall 3.32), where the proposition is a fact in the world. 3.11 and 3.12 state that a proposition is a projection of a possible fact in logical space, and the projective relation is what makes a propositional sign a representative proposition. Hence 3.13: if what is represented by a proposition is a fact in logical space, the projection does not include that fact as a fact in the world, but only the possibility of such a fact, which in turn means that there are no *a priori* propositions. All of this is neatly captured by the interpretation that thoughts are acts of geometrical projection within logical space.

A fourth central feature in the Tractarian theory that is quite consistent with the interpretation I propose is given by the relationship between the concepts of logical form and form of reality. We know that a fact can depict another fact only insofar as there is a pictorial form, which in turn is possible only if there is a logical form linking both facts. A central aspect in the doctrine of logical form is that it coincides with the form of reality, which we know is a concept closely related to logical space. Let us consider proposition 2.18 once again: “What

¹⁷ Notice that 3.1, 3.11 and 3.12 are in open agreement with the interpretation here proposed.

any picture, of whatever form, must have in common with reality, in order to be able to depict it—correctly or incorrectly—in any way at all, is *logical form, i.e., the form of reality*” (my emphasis). Logical form, form of reality and logical space are thus transcendental conditions for facts to be pictorially representative. Fogelin expresses this point in accurate and precise terms:

We must take seriously the initial identification of *logical form* with the *form of reality*. I think that what Wittgenstein is getting at is this: Every picture, of whatever kind, is a fact—a part of reality (2.141). Now just as any region of physical space can be used to represent any other, any region of *logical space* can similarly be used to represent any other. The ontology of facts was presented in the opening parts of the *Tractatus* and one important consequence of that theory was that every fact is related *in form* to every other *possible* fact. This is the underlying reason why facts have the capacity to represent other facts of an utterly diverse material quality. I think that Wittgenstein’s identification of logical form with the form of reality amounts to saying that a picture has a representational capacity simply in virtue of the form it has as a part of reality. Indeed, he seems to make even the stronger claim that, in the last analysis, all representation takes place in virtue of logical form. (Fogelin 1995, 23)

I think these remarks are quite correct. However, there is something missing. A region in logical space can represent any other region not in and by itself. For the representation to be the case, there must be a logical form, and logical form crystalizes insofar as a thought—a projection—is performed in logical space by the transcendental subject. The participation of the projecting subject is crucial for logical form, and, therefore, for representation¹⁸. That projective lines traced by the transcendental subject connect the elements in the represented facts with the elements in the projection means precisely that there is a logical form connecting both possible facts; and, as I said above, the projective lines are possible given the structure of logical space, i.e, given the form of reality. In other words, thoughts as projections illustrate the notion of logical form: projective lines—thoughts—are a display of logical form insofar as they simply follow the ‘geometric’ structure of logical space. Hence, in this reading we can clearly understand why Wittgenstein identifies the concept of logical form with the concept of the form of reality in proposition 2.18. I think this interpretation is also consistent with, and actually contributes to clarify, Wittgenstein’s own words on the matter:

3.4 A proposition determines a place in logical space. The existence of this logical place is guaranteed by the mere existence of the constituents—by the existence of the proposition with a sense.

3.41 The propositional sign with logical co-ordinates—that is the logical place.

3.411 In geometry and logic alike a place is a possibility: something can exist in it.

3.42 A proposition can determine one place in logical space: nevertheless the whole of logical space must already be given by it.

(Otherwise negation, logical sum, logical product, etc.; would introduce more and more new elements—in co-ordination).

(The logical scaffolding surrounding a picture determines logical space. The force of a proposition reaches through the whole of logical space).

¹⁸ I agree with the mentalist stance in that subjectivity plays an essential role in representation. However, I agree with the anti-mentalist standpoint in that psychology is totally irrelevant for the transcendental conditions of representation. In my proposal, the subjectivity involved is thus transcendental, not psychological.

3.5 A propositional sign, applied and thought out, is a thought.

4 A thought is a proposition with a sense.

From 3.4 we have that if there is a proposition with a sense (a projection-fact in the world), then a place in logical space (the represented-projected possible fact) is determined, so that we know with certainty that such a place exists. Similarly, 3.41 can be read as stating that a proposition, as the outcome of a thought, is a way to chart logical space as if we were using coordinates, where the coordinates indicate a place in logical space.

Proposition 3.42 is a difficult one. It is clear that a proposition can determine a place in logical space—especially if we understand it as the result of the projective method—but it is not so clear why the whole of logical space is already given by it. I think that Wittgenstein’s second parenthetical remark gives us the clue. Let us recall that logical space is the structure of reality, in the sense that it is determined by the internal properties of all objects. Now, given the internal properties of all objects, the whole structure of logical space is completely determined. Something similar holds in geometry. If the topology, the affine structure and the metric of a manifold are everywhere defined, the whole structure of the geometrical space is given, so that we know, for example, the properties of figures that can be drawn at each of its regions. Now, the logical scaffolding surrounding a picture is a region in the ‘manifold’ of logical space, and its structure is already determined and given by the internal properties of all objects. Therefore, it is also already determined and given what projective lines can be traced passing through each of those points in all and any ‘direction’ towards possible facts in logical space. Thus, the representative force of the picture at issue reaches the whole of logical space.

Finally, propositions 3.5 and 4 are, I think, especially coherent with my proposed interpretation. 3.5 can be simply read as stating that a proposition, in its representative role, is a thought in the sense that it is the result of an act of projection in logical space—a proposition is simply the outcome of a thought. Something similar holds for 4, i.e., a thought is a proposition (or any concrete picture) with a sense simply because it is a projection in logical space that determines a fact in the world that represents a fact in logical space—thoughts are projective acts that result in pictures¹⁹.

One last virtue of the interpretation I propose has to do with a difficult passage in the *Tractatus*, namely, the sequence 5.6-5.641 that deals with the subject of solipsism. This is one of the most debated topics in the literature, but there seems to be some minimal consensus about Wittgenstein intending to assert that there is something philosophically valuable in the solipsistic thesis, but that, as any other metaphysical thesis, it is a pseudo-proposition. What is the valuable content of the solipsism thesis? The answer, I think, can be grasped from the first three propositions in the section²⁰. Proposition 5.6 states that “*The limits of my language mean the limits of my world*”. Proposition 5.61 goes on to assert that

¹⁹ One may be tempted to state that since thoughts have pictures as their outcome, we could call a picture a thought, but then we would be back to the problems resulting from thoughts as facts that we found in sections 2 and 3.

²⁰ This overview of Wittgenstein on solipsism is inspired by Mounce (1989, 87-92).

Logic pervades the world: the limits of the world are also its limits.
So we cannot say in logic, 'The world has this in it, and this, but not that'.
For that would appear to presuppose that we were excluding certain possibilities, and this cannot be the case, since it would require that logic should go beyond the limits of the world; for only in that way could it view those limits from the other side as well.
We cannot think what we cannot think; so what we cannot think we cannot *say* either.

In 5.61, the identification of the limits of reality²¹, of logic, of what can be said, and of what can be thought is clear and natural, especially if we understand thoughts as projections in logical space. Logical space is the underlying structure of reality, hence the identity between the limits of logic and of reality. Projections—thoughts—occur within logical space, and the projective lines are possible and determined by the structure of logical space, hence the identity between the limits of logic-reality and the limits of what is thinkable (projectable). Finally, propositional signs are representative propositions insofar as they are the outcome of a thought-projection, hence the identification between the limits of logic-reality-thought with the limits of what can be said.

Now we can go back and try to make sense of 5.6, which asserts that the limits of *my* language are the limits of *my* reality. In 5.62, Wittgenstein moves on to assert that there is something correct in the solipsistic thesis because it is an attempt to say something that can only be shown. That reality is *my* reality, that language is *my* language, is something that is shown in every picture, but that cannot be said by any picture:

This remark [5.61] provides the key to the problem, how much truth there is in solipsism.
For what the solipsist means is quite correct; only it cannot be said, but makes itself manifest.
The world is *my* world: this is manifest in the fact that the limits of language (of that language which alone I understand) mean the limits of my world.

According to the interpretation of thoughts that I propose, they constitute projections in logical space, so that the point *P*, i.e., the transcendental subject, is a necessary condition in every thought: all projections originate in a point *P*. By definition, a thought is constituted by projective lines drawn *from* the subject. Now, to understand a proposition is to think it (3.5-4), so that the limits of logic, of reality, and of what can be said, are the limits of what *I* can think. The only source of projective lines—of thoughts—is *my* I.

These remarks allow us to refine the interpretation of thoughts as projections. There is only one point *P* from which projective lines can be drawn, but that point *P* is not defined *within* logical space, it is a boundary of logical space. This can perhaps be clarified by means of another mathematical analogy: the point *P* which is the source of all projections is a *singularity* of logical space—just as in cosmological models the Big-Bang event is a spacetime singularity,

²¹ When Wittgenstein speaks about the world (*Welt*) in 5.61, he is most certainly referring to reality (*Wirklichkeit*). I think it is clear that the appearance of 'world' is just a sloppy choice of words.

i.e., an initial boundary event that is not within the spacetime manifold²². The analogy with the visual field proposed by Wittgenstein points in that direction:

5.632 The subject does not belong to the world: rather, it is a limit of the world.

5.633 Where *in* the world is a metaphysical subject to be found?

You will say this is exactly like the case of the eye and the visual field. But really you do *not* see the eye.

And nothing *in the visual field* allows you to infer that it is seen by an eye.

Every image of my visual field shows that there is a point from which it is seen. However, that point is not a part of the visual field, so it cannot be seen. On the other hand, we know as a matter of natural science that eyes see (in the physical, physiological sense of the term), but since the point of view of the visual field cannot be seen, I cannot see that my visual field is seen by an eye. Furthermore, there may be eyes within my visual field, but I cannot see that those eyes are points of view of other visual fields, *the* visual field is always *my* visual field.

We can run the analogy also in the interpretation of thoughts as projections in logical space. Again, a projection shows that there is a point *P* from which the projective lines are drawn, but that point is not, and cannot be, a part of the projection—for it is ‘a singularity in logical space’. Lines drawn for any other point than *P* are not *projective* lines, in the sense that they do not constitute acts of thought (they cannot be *my* thoughts). Projections in logical space are always drawn from point *P*, and that point is the transcendental subject *I*. We are now in position to make sense of proposition 5.631:

There is no such thing as the subject that thinks or entertains ideas.

If I wrote a book called *The World as I found it*, I should have to include a report on my body, and should have to say which parts were subordinate to my will, and which were not, etc., this being a method of isolating the subject, or rather of showing that in an important sense there is no subject; for it alone could *not* be mentioned in a book.

Reality is always my reality, this is shown in all pictures. However, the transcendental *I* which is the source of all thoughts is not an element of reality, so nothing can be said about it—it cannot figure in any meaningful proposition. This is why the solipsistic thesis is, bottom-line, a pseudo-proposition. According to the Tractarian theory of representation, the solipsistic thesis that *only I and my representations exist* is clearly senseless (*Unsinn*), for *I* is not an element in logical space. Furthermore, if we hold fast to the reading of the solipsistic thesis that Wittgenstein proposes, i.e., as stating that the *I* to which my world ‘belongs’ is not itself a part of the world, we end up with pure realism, i.e., the view that reality is all, and only, what there is and what can be, and reality does not include the transcendental subject (5.64).

²² The precise and technical definition of a general relativistic spacetime singularity is a complex issue, but here it suffices to say that a spatiotemporal event *e* in which the spacetime curvature is infinite—its value increases without boundary as we approach *e*—is an event that is not *within* the spacetime manifold, but a boundary of the manifold—see Earman (1995, Ch. 2). The analogy is thus that the transcendental subject is to logical space as a singularity is to a spacetime manifold.

In sum, an important virtue of the interpretation of Tractarian thoughts that I propose consists in that it provides a link between the concept at issue and the difficult passage concerning the status and value of the solipsistic thesis. In proposition 5.641, Wittgenstein states that

Thus there is really a sense in which philosophy can talk about the self in a non-psychological way.

What brings the self into philosophy is the fact that 'the world is my world'.

The philosophical self is not the human being, not the human body, or the human soul, with which psychology deals, but rather the metaphysical subject, the limit of the world—not a part of it.

In simple words, the *I* that is shown in each and every picture and which constitutes a limit of reality, is the same *I* from which projective lines in logical space are drawn in each and every thought. Wittgenstein states that the transcendental self enters into philosophy insofar as the world is *my* world, I argue that the self must also enter in Tractarian philosophy insofar as it is the logical space boundary from which projective lines are traced.

Actually, it seems that we are dealing with two ways to look at the same issue. If the limits of reality, of what is thinkable, and of what can be said are the same, and since every picture shows that the world is *my* world, it is totally natural to endorse the view that the transcendental subject that is shown in every picture is the same that constitutes the point of projection in every thought. But if it is natural to identify the transcendental *I* that Wittgenstein refers to in the discussion of solipsism with the *I* that thinks, then the latter cannot be a psychological-empirical subject that builds thoughts out of psychic elements.

6. CONCLUDING REMARKS

As we saw above, both the natural and the alternative interpretation are essentially problematic. If we try to take Wittgenstein's words at face value—as in the natural interpretation—we get a view in which thoughts are logical pictures, i.e., facts, and at the same pictures whose pictorial form is purely logical. I have argued that these two statements are mutually inconsistent. If thoughts are facts, their elements are arranged in a certain concrete manner, so that its pictorial form cannot be purely logical.

On the other hand, if we drop the view that the pictorial form of thoughts is solely logical, but we retain the claim that they are pictures, and hence, facts, we must face the problem of becoming armchair scientists. The *Tractarian* theory of representation cannot depend on certain empirical (psychological) facts, so the alternative interpretation is not recommendable either.

A third interpretive maneuver is given by dropping the assertion that thoughts are facts, so that the mentioned problems are immediately avoided. However, if we do that, the essential connection between thoughts and logical form must be suitably construed. This is actually something that is achieved in the interpretation I propose. If thoughts refer to the activity of

performing geometric projections in logical space, then they are not facts, and they get essentially connected to the notions of logical form, form of reality and logical space, as I described in section 5. Projections in logical space are actually instantiations of logical form, insofar as the projective lines are possible because of the structure of logical space. Furthermore, the proposed interpretation is quite consistent with several essential tenets in the Tractarian theory of representation, and it allows a clarifying and unifying way to understand the passage about the status of the solipsistic thesis. In my proposal, it is a natural and clarifying result that the transcendental *I* that draws projective lines in logical space is the same transcendental *I* that is referred to in the discussion of the solipsistic thesis²³.

To conclude, I would only like to add that this interpretation comes at a textual cost. Even though most of the relevant propositions in which Wittgenstein deals with thoughts can be suitably interpreted, there remains one which cannot be adapted to my proposal, for its commitment to the view of thoughts as facts is impossible to avoid. The propositions in which Wittgenstein refers to *logical pictures*—most notably, the sequence 2.181-2.201 and proposition 3—can be interpreted as claiming that logical pictures are simply acts of projection in logical space, so that we avoid the conclusion that they are facts. That is, in the expression ‘logical picture’, the term ‘picture’ does not have its customary meaning, and we can then make sense of the assertion that logical pictures are thoughts, but not facts.

Similarly, even though proposition 3.01 states that “the totality of *true thoughts* is a picture of the world” (my emphasis) suggests that thoughts are pictures in a strict sense (so do 3.04-3.05), proposition 3.001 tells us that “‘a state of affairs is thinkable’ means that we can picture it to ourselves”. This last proposition is compatible with a reading of thoughts in which they refer to the *activity* of constituting pictures rather than to pictures themselves. In the German text this is even clearer: “«Ein Sachverhalt ist denkbar» heißt: Wir können uns ein Bild von ihm *machen*” (my emphasis). Thus, we can suitably understand most of Wittgenstein’s propositions that refer to thoughts as pictures: they simply refer to the outcome of thoughts. The totality of true thoughts referred to in 3.01 can be simply understood as the totality of true pictures that result from thoughts.

So far, so good. However, I must admit that there is one proposition that cannot be adapted to my proposal. Proposition 3.2 states that “in a proposition a thought can be expressed in such a way that the elements of the propositional sign correspond to *the objects of the thought*” (my emphasis)—and it is clear that this statement connects to Wittgenstein’s remarks in his epistolary discussion with Russell. In the interpretation I have introduced, thoughts cannot be composed by objects, for they are not facts. The proposition at issue, however, undeniably takes a thought as a fact composed by objects.

I think that proposition 3.2 is an indication that, as it is literally presented, the Tractarian doctrine of thoughts is unavoidably problematic. Since this proposition is openly committed

²³ One may complain that my proposal is more a metaphor than an outright explanation. I concede that it is a metaphor. However, I think it is explanatory in the same sense that the very notion of logical space—a metaphor as well—is explanatory with respect to the ontology that works as a condition for the possibility of pictorial representation. Anyhow, a recent work (Pilch 2017) construes logical space in non-metaphoric terms, i.e., as a concrete mathematical structure based on the Tractarian theory of truth-functions. An interested development of my interpretation would be to explore how to embed it within Pilch’s proposal.

to the view of thoughts as facts, the problem of *a priori* neuroscience certainly comes up, and the problem of factual pictures with pictorial form which is strictly logical may come up as well.

I think that the description of thoughts as facts is just untenable. One more way to see this is that if thoughts are made up of psychological objects, it is quite natural to assume that the *I* that produces and possesses those thoughts is an empirical subject. But this is inconsistent with Wittgenstein's discussion about the status and meaning of the solipsistic thesis. Such a thesis tells us that the self that is shown in every picture and in every thought, and that constitutes a limit of the world and a limit of what is thinkable, it is not an element in the world. Thus, the mentioned self must be the same self that thinks, for the limits of reality are the same limits of what is thinkable. Thus, we can conclude that Wittgenstein's endorsement of a description of thoughts as facts must be jettisoned.

REFERENCES

- Anscombe, G. E. M. 1965. *An Introduction to Wittgenstein's Tractatus*. New York: Harper & Row.
- Black, M. 1964. *A Companion to Wittgenstein's 'Tractatus'*. Cambridge: Cambridge University Press.
- Earman, J. 1995. *Bangs, Crushes, Whimpers, and Shrieks: singularities and acausalities in relativistic spacetimes*. New York & Oxford: Oxford University Press.
- Fogelin, R. 1995. *Wittgenstein*. London & New York: Routledge.
- Hacker, P. M. S. 1986. *Insight and Illusion: themes in the philosophy of Wittgenstein*. Oxford: Oxford University Press.
- Hacker, P. M. S. 1990. *Wittgenstein: meaning and mind*. Oxford: Basil Blackwell.
- Hamilton, K. 2001. "Wittgenstein and the Mind's Eye." In *Wittgenstein: biography & philosophy*, edited by J. Klagge, 53-97. Cambridge: Cambridge University Press.
- Kenny, A. 2006. *Wittgenstein*. Oxford: Blackwell Publishing.
- Malcolm, N. 1997. *Wittgenstein: a religious point of view?* London: Routledge.
- Maudlin, T. 2012. *Philosophy of Physics: Space and Time*. Princeton: Princeton University Press.
- Mounce, H. O. 1989. *Wittgenstein's Tractatus: an introduction*. Oxford: Basil Blackwell Publisher.
- Pilch, M. 2017. «The Structure of Wittgenstein's Logical Space.» *Wittgenstein Studien* 8: 15-60.
- Schroeder, S. 2006. *Wittgenstein: the way out of the fly bottle*. Cambridge: Polity Press.
- Stenius, E. 1996. *Wittgenstein's Tractatus: a critical exposition of the main lines of thought*. Bristol: Thoemmes Press.

- Torretti, R. 2016. «Nineteenth Century Geometry.» In *Stanford Encyclopedia of Philosophy*.
Edited by E. Zalta.
- Wittgenstein, L. 1961. *Notebooks 1914-1916*. Edited by G. H. von Wright and G. E. M.
Anscombe. Translated by G. E. M. Anscombe. New York: Harper & Row.
- . 2002. *Tractatus Logico-Philosophicus*. Translated by D. P. Pears and B. F. McGuinness.
London & New York: Routledge.