SCIENTIFIC UNDERSTANDING IN ASTRONOMICAL MODELS FROM EUDOXUS TO KEPLER

In the following essay I present a narrative of the development of astronomical models from Eudoxus to Kepler, as a case-study that vindicates an insightful and influential recent account of the concept of scientific understanding. Since this episode in the history of science and the concept of understanding are subjects to which Professor Roberto Torretti has dedicated two wonderful books—*De Eudoxo a Newton: modelos matemáticos en la filosofía natural* (2007), and *Creative Understanding: philosophical reflections on physics* (1990), respectively—this essay is my contribution to celebrate his outstanding work and career in this volume. I dedicate this piece to Roberto, dear friend and mentor, in gratitude for all his inspirational work and personal support, which has greatly helped me, and many others, to better understand that human wonder we call scientific knowledge.

1. Scientific Understanding

The concept of understanding was neglected by contemporary philosophy of science for a long time. Carl Hempel's (1965, 425-433) is a paradigmatic example of the stance adopted by philosophers influenced by logical positivistic principles. In the context of his deductive-nomological model of explanation, he described understanding as a psychological *a-ha!* experience that accompanies scientific explanations. If we consider it as a psychological byproduct of explanations, whether a subject S obtains understanding from an explanation or not, and the type of understanding that S gets from the explanation, depends crucially on subjective and context-dependent factors, such as S' expectations, motivations, background knowledge, etc.

Despite its subjectivity, Hempel did acknowledge an epistemic dimension to understanding. He stated that a phenomenon is understood if its occurrence is *expected*, given the corresponding laws of nature and the initial conditions. Thus, the epistemic dimension of understanding reduces to the notion of explanation. Consequently, given its subjective nature and the subsumption of its epistemic import under the concept of explanation, and following the logical empiricist stance that only the logical aspects of the connection between evidence and scientific explanations are philosophically relevant, Hempel affirmed that understanding is not a concept worth of philosophical inquiry.

Despite the demise of logical empiricism, the proscription of understanding from philosophical consideration continued. At most, related issues like the type of intelligibility of nature that scientific theories convey were treated during the 60s and 70s in the context of discussions about different models of explanation. In the 80s, with the introduction of approaches that made clear the importance of pragmatic issues for explanation, the environment became less hostile for a serious philosophical consideration of understanding.[1] However, although van Fraassen's (1980) pluralistic and pragmatic account recognizes that contextual and subject-based factors are essential for scientific explanation—opening the door for a consideration of understanding *vis à vis* explanation—this approach states that explanation (and consequently understanding) is a pragmatic, but not an epistemic, goal of science.

During the last two decades, though, proposals have been introduced that consider understanding as an essential epistemic goal of science, and as crucial factor in its practice, recognizing its intrinsically pragmatic

[1] For a conceptual overview of philosophical stances on scientific explanation, see Woodward (2014).

nature.[2] In his celebrated *Understanding Scientific Understanding*, Henk de Regt (2017) proposes an illuminating account in which understanding is a basic epistemic goal of science, is pragmatic, and is not reduced to explanation. He starts out from a distinction between three senses of the concept of understanding in science. First, we get understanding *of a phenomenon* (UP) when we have an adequate scientific explanation of it. Now, UP is usually accompanied by a subjective psychological experience, which de Regt calls the *phenomenology* of understanding (PU). UP and PU can be recognized in Hempel's received view, that is, understanding is a subjective experience (PU), and it is conveyed by scientific explanations (UP). However, a third sense, understanding *a theory* (UT), must also be acknowledged. By UT, de Regt refers to *the ability to use* a theory, and he evaluates it as a necessary condition for UP.

De Regt claims that scientific explanations are typically given by models (in a very wide sense of the term) built out of theories. Explanatory models represent the phenomena, obeying the constraints imposed by the corresponding theory: models mediate between theory and target phenomena. Following Morgan and Morrison (1999) and Cartwright (1983), de Regt points out that the construction of models is not an algorithmic or merely deductive exercise—scientific theories do not come with a recipe for model construction. This process usually involves idealizations and abstractions, and it requires skills, good sense, and judgment on the scientist's part. Thus, the construction of explanatory models requires the ability to use the corresponding theory.

De Regt connects this ability to the notion of an *intelligible theory*. A theory T is intelligible for a scientist S if S is able to build explanatory models out of T. In de Regt's own words, intelligibility is "the value that scientists attribute to the cluster of qualities of a theory [...] that facilitate the use of the theory" (2017, 40). Intelligibility is then a *relational* property, it depends both on the skills and background knowledge of S, and on the qualities of T that fit S' skills.

Hence, the intelligibility of a theory is in a sense subjective: although independent of PU, the intelligibility of T essentially refers to features of S. However, this subjective dimension does not affect the objectivity of science. Toolkits of intelligibility are acquired and developed by scientists within a community. Thus, there exist public criteria and standards that establish if the lines of reasoning followed by individual scientists in the construction of explanatory models conform to objectivity conditions. Besides, intelligibility, understood as a value, must also conform to the basic values of empirical adequacy and internal consistency—following Longino (1990) and Douglas (2009), de Regt supports a conception of scientific objectivity in which values, including intelligibility, play an essential and constitutive role.[3]

Now, this framework of objectivity leaves plenty of room for the variation of intelligibility standards, both synchronically among scientific (sub)communities, and diachronically across the history of science. That is, certain skills, and the theories that conform to such skills, are rendered as tools for intelligibility and as intelligible, respectively, *relative to a specific context*. Different skills and different types of theories are valued as intelligible by scientists of different communities and/or different times. In other words, intelligibility is a pragmatic concept.

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[2] Torretti (1990) is an early approximation to understanding as a central goal of science. For an overview of the different contemporary stances on understanding, see de Regt and Baumberger (2019). See also the articles in de Regt, Leonelli, and Egner (Eds.) (2009), and in Grimm, Baumberger, and Ammon (Eds.) (2017).

[3] In this conception of scientific objectivity, empirical adequacy and internal consistency are also values. That is, although both are basic values that transcend revolutionary changes in the history of science, there are always contexts in which they can be traded for other values. As we will see below, scientific understanding can be obtained from false theories. For a treatment of inconsistency in scientific theories, see Vickers (2013) and Frisch (2014).

Given the definitions presented, UT and intelligibility are necessary conditions for UP. Explanatory models can be generated from a theory insofar as the corresponding theory is understood and intelligible (UT). Thus, scientific understanding in the sense of UT is an essential epistemic aim of science, and given the characterization offered by de Regt, it is at the same time pragmatic. Furthermore, there is a virtuously circular interconnection between UP and UT. Scientists use intelligible theories to build explanations, which may turn out successful or not. Success does not depend only on the intelligibility of the theory, but also, and crucially, on the basic values of empirical adequacy and consistency.[4] Now, if the explanations are indeed successful, the skills and forms of intelligibility associated to the theory get vindicated also as providing understanding *of the phenomena*. As a result, those skills and the corresponding qualities of theories can get canonized as paradigms of UT and UP, and applied in subsequent scientific inquiry.

The essential link between understanding and explanation, incarnated in the interconnection between UT and UP, is captured by de Regt's definition of a criterion for understanding a phenomenon (CUP): "a phenomenon P is understood scientifically if and only if there is an explanation of P that is based on an intelligible theory T and conforms to the basic epistemic values of empirical adequacy and internal consistency" (2017, 92). Now, as we said, a theory is intelligible if scientists are able to use it, and this ability depends on skills and background knowledge possessed by the scientist, and also on the qualities of the theory that adapt to such skills. Given the pragmatic nature of UT, the skills and the qualities of theories that are considered as paradigms and/or norms of intelligibility and understanding vary across communities and historical periods. De Regt formulates a criterion for intelligibility of theories (CIT) that openly acknowledges its context-dependency, and that makes room for diachronic and synchronic variation: "a scientific theory […] is intelligible for scientists (in context C) if they can recognize qualitatively characteristic consequences of T without performing exact calculations" (2017, 102).

De Regt offers several case-studies in the history of physics that support his views. Examinations of episodes in the development of Newtonian gravitation, electrodynamics, statistical mechanics and quantum mechanics provide evidence for the diachronic and synchronic variability of standards of intelligibility, supporting in turn de Regt's pragmatic account of scientific understanding. The variation is of course multifactorially determined, but the success and failure of (types of) theories that conform to different criteria of intelligibility are always crucial. Features like visualization, causality, mechanisms, and mathematical abstraction, come and go as canons of intelligibility of theories (UT), and of the understanding of phenomena (UP), depending on the success or failure of the theories that incarnate those features.

We will now add another case-study that also vindicates de Regt's proposal: the development of astronomical models from Eudoxus to Kepler. As we will see, this crucial and long episode in the history of physics shows with special clarity that UT is a condition for UP, and also the pragmatic nature of scientific understanding.

2. Astronomical Models from Eudoxus to Kepler[5]

The development of astronomical geometric models from Eudoxus to Kepler is a crucial stage in the constitution of modern physics. This episode in the history of science spans about two millennia. Eudoxus

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[4] As we will see below, false models can be successful, and therefore convey UP.

[5] For the history of astronomy from Eudoxus to Kepler, see Barbour (2001), Crowe (2001), Dijksterhuis (1986), Dreyer (1953), Evans (1998), Jacobsen (1999), Koyré (1973), Kuhn (1995), Linton (2004), Neugebauer (1975; 1983), and Torretti (2007). Most of the figures below are based on Linton (2004).

of Cnidus, in the IV century BC, was the first astronomer to invent a comprehensive geometric model that accounts for naked-eye celestial phenomena. Eudoxus' model strictly follows an aesthetic-metaphysical principle of circular uniform motion for celestial objects. About 500 years later, Claudius Ptolemy, elaborating on ideas introduced earlier by Apollonius and Hipparchus, invented a much more refined and empirically adequate model that would dominate astronomy for about 13 centuries. Although Ptolemy's astronomy was also based on circular motion, it violated the uniformity of celestial motion. A serious rival to Ptolemy's model would only be formulated by Nicolaus Copernicus in 1543. Although the introduction of a heliocentric model was indeed a major innovation, Copernicus' model was actually rather conservative: one of its main motivations was the reintroduction of the ancient principle of circular uniform motion. Four decades after the work of Copernicus, Tycho Brahe proposed a model that captured the advantages of Copernicus' over Ptolemy's, but that reinstalled the Earth as the center of the universe. Finally, in 1609, Johannes Kepler, following a truly revolutionary insight, amended Copernicus' model. The resulting system of circular (but not uniform) motion, however, was doomed to fail in the representation of the orbit of Mars by an unacceptable observational margin. Since Kepler also showed that Copernicus', Brahe's and Ptolemy's models were geometrically inter-translatable, the failure of his improved version of Copernicus model also showed the failure of its two rivals. Thus, Kepler's work signed the ruin of the project of circular motion astronomy, and opened a new path in the development of physics with his three laws of planetary motion. As we will see in section 4, the following narrative of the rise and fall of circular motion astronomy constitutes a historical case that clearly illustrates the pragmatic character of scientific understanding, and its relevance and necessity for scientific explanations.

2.1. Naked Eye Astronomy

The astronomical models we will review were designed to account for celestial phenomena that can be observed with the naked eye. The telescope, in the hands of Galileo, only enters the stage of science at the beginning of the 17th century. Such phenomena were basically four: the motion of the stars, the Sun, the planets, and the Moon. The representation of the Moon's motion is the most complex element in all the models to be considered, so for brevity and simplicity, we will not treat it here.

Looking at the sky for a few consecutive nights, we see that the stars remain stationary with respect to each other, and, if we stand somewhere in the Northern Hemisphere, we also see that they describe a counterclockwise circular path around a fixed point.[6] The farther a star with respect to the fixed point, the larger the circle it describes (see Figure 1). These observations suggest that the stars lie fixed on a sphere that rotates westwards, with an axis that passes through the fixed point and the center of the Earth (defining the celestial north and south poles), with a period of 23 hours and 56 minutes (the *sidereal* day).[7] The plane perpendicular to the axis is called the celestial equator (see Figure 2). This *celestial sphere* was taken to be the outermost limit of the universe, and this spherical image was adopted in all the astronomical models we will review.

[6] Another fixed point around which the stars are seen to rotate clockwise is observed from the Southern Hemisphere. The main characters in our story were inhabitants of the Northern Hemisphere, so we will adopt their perspective.

[7] The sidereal day must be distinguished from the *solar* day: the time it takes the Sun to return to the same local meridian (the time between consecutive noons). The solar day thus defined is variable along the year. Using the idealized mean Sun (see below for the distinction between the real and the mean Sun), the solar day can be defined to last 24 hours.

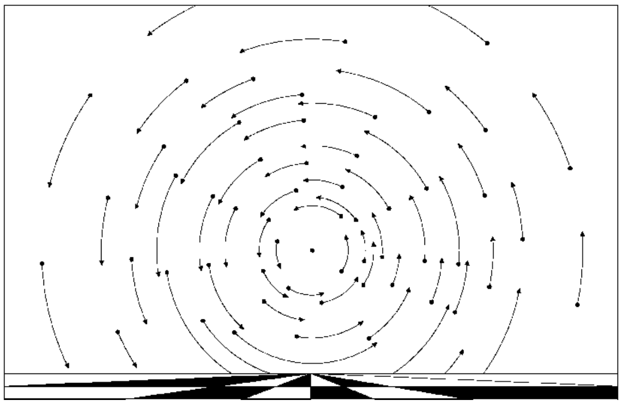

Fig. 1. The stars at night

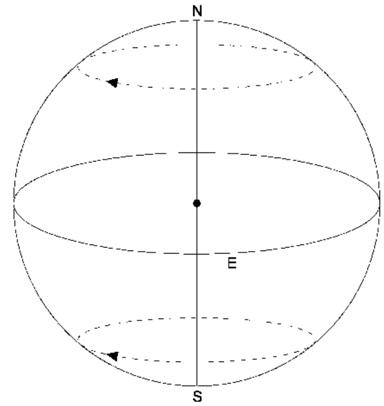

Fig. 2. The celestial sphere and the equator E

Like the stars, the Sun is also observed to describe a diurnal circular motion. It rises somewhere in the east, describing an arc in the sky until it sets somewhere in the west, to rise again somewhere in the east the next day. As it can be seen in Figure 3, the observed angle of the daily arc with respect to the horizon depends on the terrestrial latitude. The Sun also displays another apparent motion. The point at which it rises on the east and at which it sets in the west is not the same every day. At two particular days, it rises almost exactly at east and sets almost exactly at west. When that happens, day and night last almost the same everywhere on the Earth, which is why we call those days equinoxes (from Latin for *equal night*). Then the Sun rises and sets at points that progress northerly or southerly, reaching the northmost and southmost rising and setting points about three months after an equinox. The days when the Sun reaches its north or southmost points for rising and setting are called solstices. At the solstices, the day is the longest or shortest of the year, depending on which hemisphere one is standing on. After a solstice, the same process reverses, reaching an equinox again about three months later. The whole cycle lasts a year.

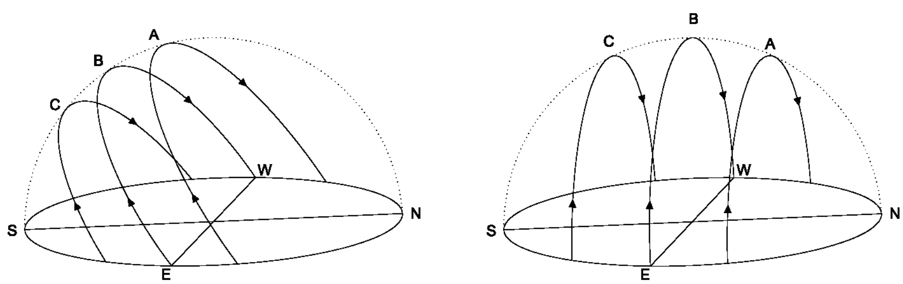

Fig. 3. Diurnal motion of the Sun at equinoxes (B) and solstices (A, C), as seen standing about 50° north (left), and at the equator (right)

If we consider this yearly motion in connection with the motion of the celestial sphere, we have that the Sun moves with respect to the background stars in a particular way. That is, if we determine the position of the Sun with respect to the celestial sphere over successive sidereal days, we get that it has moved every time a little bit. If we look at a series of such positions, we see that the Sun describes a great circle along the celestial sphere, as represented in Figure 4. The plane described by this great circle is called the *ecliptic*, and it is inclined with respect to the celestial equator by an angle of about 23°40'. This geometric arrangement allows us to define the year, the equinoxes, and the solstices more precisely. The equinoxes are the two points in which the ecliptic intersects the celestial equatorial plane, and the solstices are the points in the ecliptic with maximum and minimum celestial latitude (±23°40'). The year is the time it takes the Sun to complete the ecliptic circle.

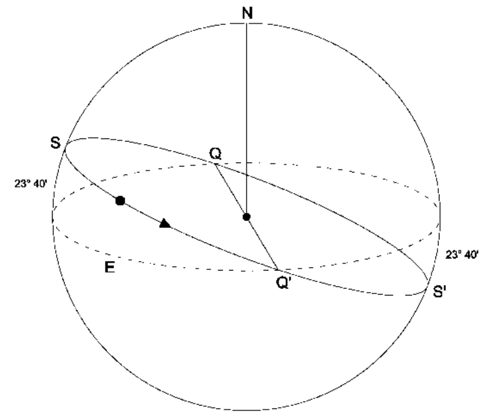

Fig. 4. The ecliptic plane SQ'S'Q, determined by the solstices S and S', and the equinoxes Q and Q', inclined 23°40' with respect to the equatorial plane E

The planets that can be seen with the naked eye are Mercury, Venus, Mars, Jupiter and Saturn.[8] They all display a diurnal motion, that is, they rise and set in the horizon every night. They also exhibit a motion similar to the yearly motion of the Sun. If we look at the position of a planet with respect to the background stars over successive sidereal days, it also moves slowly. However, the planets do not describe a great circle in the celestial sphere as the Sun does. Concerning their celestial longitude, the apparent motion of the planets is such that they move eastwards along their trajectory with respect to the background stars, except for periods in which they gradually turn around westwards, to later resume their usual eastward direction. This phenomenon, represented in Figure 5, is called *retrograde motion*. As it can be seen in Figure 6, the planets have a variable celestial latitude, but they are never far from the ecliptic, they always lie within a belt along the celestial sphere, bisected by the ecliptic, of about 16° wide. Each planet has a characteristic zodiacal period, i.e., the average time it takes the planet to make full round around the zodiac, and a characteristic synodic period, i.e., the average time between periods of retrograde motion (see Table 1).

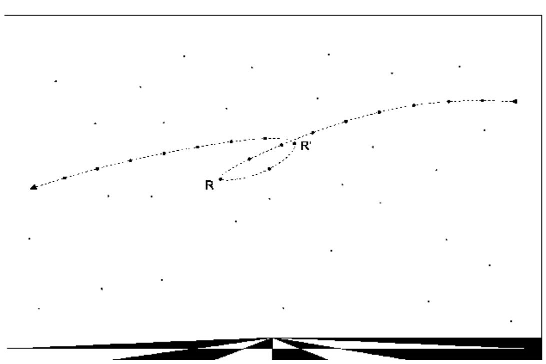

Fig. 5. Retrograde motion of a planet, between positions R and R'

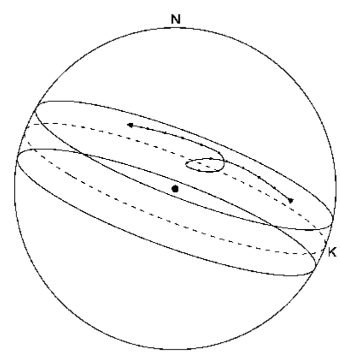

Fig. 6. A piece of the motion of planet along the zodiac belt K, including retrograde motion

[8] Since the observed motion of the Moon, Mercury, Venus, the Sun, Mars, Jupiter and Saturn is not as patently regular as the motion of the stars, the ancient Greeks named them '*planetai*', i.e., wanderers.

2.2. Eudoxus' Concentric Spheres

The first elaborate geometric astronomical model that aims to represent the described phenomena is due to Eudoxus of Cnidus (390 BC – 337 BC). Although his works are lost, we know the essentials of the model through Aristotle's (1924) *Metaphysics* and Simplicius' (2005) commentary (VI century AD) on Aristotle's (1922) *On the Heavens*.

Eudoxus' representation of the universe is a system of spheres, all centered in the Earth. The observable diurnal motion of the stars is represented by a single sphere that rotates westwards around the axis defined by the celestial north and south poles, with a period of a sidereal day—this is just the celestial sphere in Figure 2. The observed motion of the Sun is represented by the combined motion of three spheres. As it is shown in Figure 7, the Sun's first sphere moves exactly as the sphere of the fixed stars. The second sphere rotates eastwards, with a period of a year. Its axis of rotation is inclined 23°40' with respect to the first sphere's axis, and its endpoints are fixed on two antipodal points in the first sphere.[9] Placing the Sun in a suitable fixed point in the equator of the second sphere, its observed motion is reproduced by the model.[10]

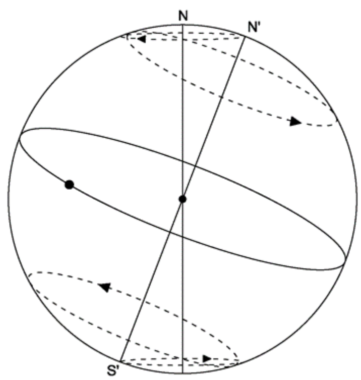

Fig. 7. NS is the axis of rotation of the Sun's first sphere, N'S' the axis of its second sphere

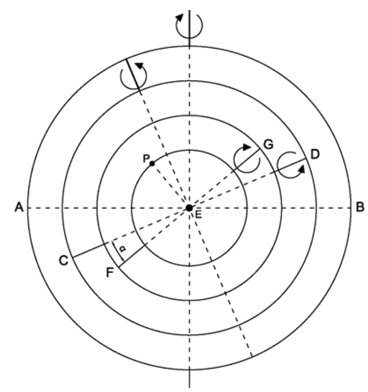

Fig. 8. Eudoxus model for a planet. AB is the equator, CD is the ecliptic, the planet is at P.

Eudoxus assigned 4 spheres to each planet, as shown in Figure 8. The first sphere reproduces the diurnal motion, so it is just like the sphere of the distant stars. The second reproduces the eastward motion of the planet along the zodiac. Just like the Sun's second sphere, its axis is inclined with respect to the axis of the first one by an angle of 23°40'.[11] Its period of rotation is the zodiacal period of the planet. The third and fourth spheres take care of retrograde motion. The third sphere's axis CD lies on antipodal points in the equator of the second sphere, that is, it lies on the ecliptic. The axis FG of the fourth sphere is slightly

[9] The numerical values for the parameters in Eudoxus' model are contemporary reconstructions. Neither Aristotle nor Simplicius included precise values in their mostly qualitative explanations of the model.

[10] Eudoxus included a third sphere, with its axis inclined a small angle with respect to the second sphere's axis. It is generally affirmed that he wrongly attributed the Sun a small latitudinal motion with respect to the ecliptic. However, Linton (2004, 28) affirms that in Eudoxus' times the ecliptic was vaguely defined as some great circle within the zodiac in the celestial sphere. The definition of the ecliptic in terms of the motion of the Sun, Linton states, was introduced about two centuries later by Hipparchus. If this is correct, Eudoxus' third sphere was justified. Support for this interpretation (see Neugebauer 1975, 633) lies on the fact that an observational determination of the precise path of the Sun with respect to the background stars was very difficult, so an estimation of a $1/15$ part of a circumference, i.e., 24°, naturally suggests. Once established this pseudo-ecliptic plane, more precise observations of the path of the Sun would lead to the introduction of some latitudinal motion with respect to the pseudo-ecliptic.

[11] Or perhaps 24°, see footnotes 9 and 10.

inclined with respect to the axis of the third sphere by an angle α, with a specific value for each planet. The third and fourth spheres rotate in opposite directions, both with a period given by the synodic period of the planet. The combined motion of the third and fourth spheres produce loop-shaped figures, called *hippopedes*, representing retrograde motion, and the motion of the second sphere drags the hippopedes along the planet's zodiacal path. Table 1 shows the values of zodiacal and synodic periods.

	Zodiacal period in years	Synodic period in days
Mercury	1	115
Venus	1	584
Mars	1,88	780
Jupiter	11,86	399
Saturn	29,46	378

Table 1. Zodiacal and synodic periods of the planets

Eudoxus' model says nothing about the order of distances of the planets with respect to the Earth—the radii of the spheres play no role. Actually, for the model to work, we do not need to take them as real physical entities in any sense, but only as geometric-kinematic configurations that represent the motions of the planets (cf. Torretti 2007, 40). However, ancient astronomers invoked arguments that mixed observations and a lucky guess in order to establish that order. Observations clearly show that the Moon is the closest celestial object: its observed size, and the fact that along its trajectory it covers other planets and the Sun (as in solar eclipses). As for the rest of the planets, the distance of a planet to the Earth was given by the zodiacal period—the longer the zodiacal period, the farther the planet from the Earth. Thus, Saturn, Jupiter and Mars are farther than the Sun. For this reason, these were known as the *superior planets*. The Sun, Venus and Mercury, on the other hand, do not differ in their zodiacal periods, so their order was a controversial issue. However, after Ptolemy's work, consensus was reached and Mercury and Venus, in that order, were considered to be closer to the Earth than the Sun—and for this reason they were known as the *inferior planets*.

The kinematic behavior of the inferior planets differs from the superior ones in an important aspect. The maximum elongation (angular distances as seen from the Earth) between the Sun and Mercury and between the Sun and Venus, are, respectively, 29° and 47°. This means that, in terms of their elongation, inferior planets are never far from the Sun, so they can only be seen near sunset or dawn. This also explains why the zodiacal periods of Venus and Mercury are equal to the solar year (see table 1). On the other hand, the elongation between the Sun and the superior planets can go up to values near 180°, so the Earth and superior planets can be in *opposition*. Now, although the Earth and inferior planets can never be in opposition, both superior and inferior planets are in *conjunction* when the elongation angle takes minimum values (see Figure 9).[12] Retrograde motion of superior planets is always observed near opposition, whereas for inferior planets it is always observed near conjunction.[13] In Eudoxus' model, both features are coincidences. The system of

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[12] Elongation at opposition is not exactly 180º, and elongation at conjunction is not exactly 0º, due to the fact that along their orbits of planets show a small latitudinal distance from the ecliptic (although they always stay within the zodiac).

[13] From a heliocentric perspective, for interior planets a distinction between *superior* and *inferior* conjunction can be traced. An interior planet is in inferior conjunction when it lies between the Earth and the Sun, and in superior conjunction when the Sun lies between the planet and the Earth. The retrograde motion of Mercury and Venus is observed

concentric spheres does not enforce that there is a maximum angle of elongation for inferior planets, nor a connection between retrograde motion and conjunction or opposition.

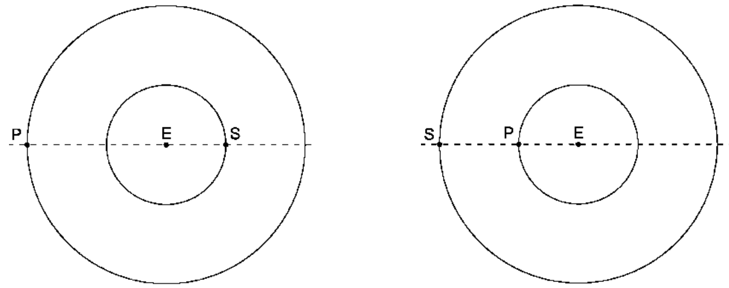

Fig. 9. A superior planet P in opposition (left),
and an inferior planet in conjunction (right)

There are several shortcomings in Eudoxus' model. First, the latitude of planets with respect to the ecliptic cannot be correctly reproduced—according to Simplicius, the representation of celestial latitude was incorrectly taken care of by the width of the hippopedes. Second, the hippopedes resulting from the third and fourth sphere of each planet always have a loop ∞-shape, whereas some of the observed retrograde motions are *z*-shaped. Third, variations in the apparent size of the Moon and some planets could not be accounted for—in Eudoxus' model, a celestial object is always equidistant from the Earth, so its apparent size should not change. Finally, and most importantly for the historical development of astronomy, the motion of the Sun along the ecliptic is not strictly uniform. We can divide the ecliptic circle in four equal arcs of 90º, subtended by the solstices and equinoxes. The Sun covers these four arcs in different periods—which means that the seasons of the year are not equally long. A similar feature holds for the planets, for their motion along the zodiac occurs at variable angular velocities. But since in Eudoxus model all spheres rotate uniformly, these variations could not be accounted for.

Despite these issues, Eudoxus' model set the heuristic principles to study the motions of the heavens in subsequent astronomical geometrical models, i.e., the apparent irregularities in celestial motions were to be explained in terms of the combination of several circular uniform motions:

> The details of Eudoxus' theory are not known with any certainty, but we do know that the scheme exerted a profound influence over the development of astronomical thought. […] This was because it demonstrated the power of geometrical techniques, in that superpositions of simple uniform rotations could be used to model extremely complex behavior. (Linton 2004, 32)

2.3. The Metaphysics of Circular Uniform Motion

The explanation of the motion of celestial objects in terms of circular uniform motion was not just a matter of conceptual and empirical convenience. Since the early days of Greek philosophy, and following a Pythagorean tradition, the idea that this type of motion is essentially appropriate to the heavens was a central metaphysical principle in the examination of nature. In his dialogue *Timaeus*, Plato (2000) claimed that the creation of the heavens responded to the demiurge's decision to introduce time in nature, so circular uniform motion—associated to eternity and strict regularity—was the natural choice for fulfilling this task. This is

near inferior conjunction. From a geocentric perspective, as in Eudoxus' model, this distinction cannot be traced, for the Sun cannot lie between an inferior planet and the Earth.

just an example that metaphysical and aesthetic considerations grounded the value that the ancients assigned to circular uniform motion. Eudoxus' was a member of the Academy, so Simplicius was probably right when he states that his model was a response to the challenge set by Plato of finding a representation of the motion of the planets in terms of circular uniform motion:

> Eudoxus of Cnidus is said to be the first of the Hellenes to have made use of such hypotheses, Plato (as Sosigenes says) having created this problem for those who had concerned themselves with these things: on what hypotheses of uniform and ordered motions could the phenomena concerning the motions of the planets be preserved? (Simplicius 2005, 33)

In *Physics* and *On the Heavens*, Aristotle (1936; 1922) further elaborated on the importance of circular uniform motion in astronomy. He claimed that the planets (in the ancient sense of the term, see fn. 8) and the stars are made of an element, different of the four elements constituting terrestrial bodies (earth, water, wind and fire), called *ether*. By essence, circular uniform motion is the goal (*télos*) that corresponds to bodies made of this element. Aristotle's conception of celestial objects is thus teleological: the planets and the stars move uniformly in circles not because they are forced to, but because that motion constitutes their essential *télos*.

The Earth is at rest in Aristotle's physics. His central argument against terrestrial motion was that if it rotated, a body thrown straight upwards would fall back to the ground some distance to the west with respect to the place from which it was thrown, against what is observed—and a similar argument can be run against any kind of displacement of the Earth. In Aristotle's terrestrial physics, the *télos* for the elements earth and water is to reach their natural place: the center of the universe. This provides a qualitative-teleological explanation of why the Earth is located precisely at the center of the universe, and why heavy objects fall. Without anything like a concept of inertia, the view of an immobile Earth was the most reasonable, and it remained unchallenged until the 16th century.

Eudoxus' model is a nice geometric fit for Aristotle's physics. It is coherent with an immobile central Earth, with the motion of the stars around it, and it provides an explanation of retrograde motion in terms of uniform circular motion—all the concentric spheres rotate uniformly. In Aristotle's interpretation of Eudoxus' model, the spheres are physical and made of ether, not only kinematic configurations.

2.4. Ptolemy's Model

The most important astronomers between Eudoxus and Ptolemy are Apollonius of Perga (3rd century BC) and Hipparchus of Nicaea (ca. 190 BC – ca. 120 BC). Apollonius showed that the problem of the variable angular speed of the Sun along the ecliptic could be solved by a circular uniform orbit not centered on the Earth. In Figure 10, the orbit of the Sun is represented by the dashed circle with radius DS, whose center D is at a distance ED from the Earth E—ED is the *eccentricity* of the Sun's orbit. The angular distance between equinoxes and solstices as seen from the Earth is 90°, but given the eccentricity, they subtend slightly different angles from D, so the determined arcs have slightly different lengths Thus, exactly because the Sun S moves uniformly in a circle centered in D, it covers the seasonal arcs in the ecliptic in different periods.

A crucial contribution by Apollonius was that he showed that a different mathematical model can achieve the same result. In Figure 10, consider now the solid circle centered on the Earth with radius EC, the *deferent*. Let C move uniformly anticlockwise along the deferent, and define a circle centered in C with radius CS, the *epicycle*. Let S move uniformly and clockwise along the epicycle, but with the same angular speed as C along the deferent, so that $EDSC$ is always a parallelogram. It is clear that the position of S in the eccentric orbit model is always the same as the position of S in the deferent-epicycle model, so both models reproduce the motion of the Sun equivalently.

The deferent-epicycle model, Apollonius also showed, can be used to represent retrograde motion. As figure 11 illustrates, by letting the epicycle rotate in the same direction as the deferent, but with a different angular speed, the trajectory of a planet fixed in a point in the epicycle is the dashed line, which contains regular periods of retrograde motion.

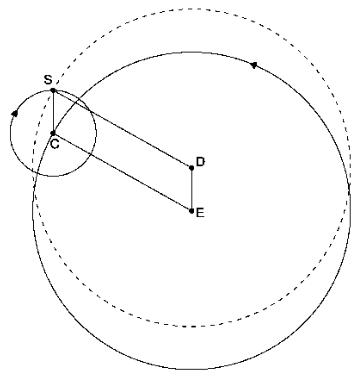

Fig. 10. Apollonius' eccentric solar orbit, and its equivalent model of deferent and epicycle

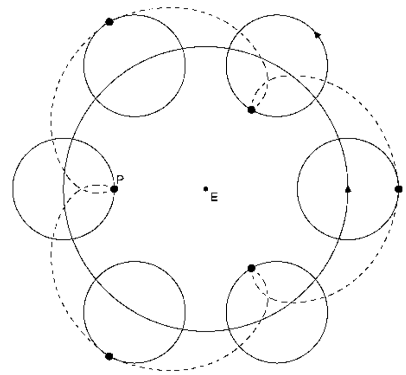

Fig. 11. Apollonius' method of deferent and epicycle for retrograde motion

In order to put Apollonius ideas to work in a model that saves the phenomena, the value of several parameters must be solved mathematically and determined empirically. In the case of the Sun observed motion (see Fig. 10), the direction of ED and its relative length with respect to DS must be established, through the determination of the angle DES. This requires precise observations and mathematical calculations that without modern trigonometry are not trivial.

Hipparchus did just that, and his method is illustrated in Figure 12. P_1, P_2, P_3 and P_4 are the positions of the Sun at the vernal equinox, the summer solstice, the autumnal equinox and the winter solstice, respectively. All four points can be determined observationally, and Hipparchus measured that the Sun travels from P_1 to P_2 in 94 ½ days, and from P_2 to P_3 in 92 ½ days. To determine the direction of the eccentricity EO, we must determine the angle $\lambda = P_1EA$, where A is the apogee. Apogee and perigee are the *apsides*, i.e., the points in which the Sun is farthest and closest to the Earth in its eccentric orbit, respectively. If we determine the angle λ, we get the line joining the apsides, and EO lies on it. With some modern trigonometry (see Linton 2004, 56), we obtain $\tan\lambda = \sin\alpha/\sin\beta$ and $EO/OS = \sin\alpha/\sin\gamma$. To determine the values of α and β we can use the constant angular sped of the Sun along its orbit, namely, $w{\sim}59'8''$ per day ($360°$ per $365\ 1/4$ days).[14] Then, $\lambda = 65°\,25'39''$. To calibrate the model to observations, we also need the ratio between the eccentricity and the radius of the Sun's orbit. With modern trigonometry we get $EO/OS = 1/24{,}17$. Without modern trigonometry and following a tortuous mathematical method, Hipparchus obtained $\lambda = 65°\,30'$ and $EO/OS = 1/24$. Notice that these results are necessary to calibrate the deferent epicycle model of the Sun's orbit as well: with EO/OS we get the required ratio between the epicycle radius

[14] $\angle P_1OP_2 = \alpha + \beta + 90°$ and $\angle P_2OP_3 = \alpha - \beta + 90°$, so that $\angle P_1OP_3 = 2\alpha + 180°$. The Sun goes from P_1 to P_2 in $94\ ½ = 189/2$ days, and from P_1 to P_3 in 187. Thus, $\alpha + \beta + 90° = 189w/2$, and $2\alpha + 180° = 187w$, solving the last two equation we get α and β.

and the deferent radius, and the direction of EO (given by the angle λ) is necessary to build the parallelogram $EDSC$ in figure 10.

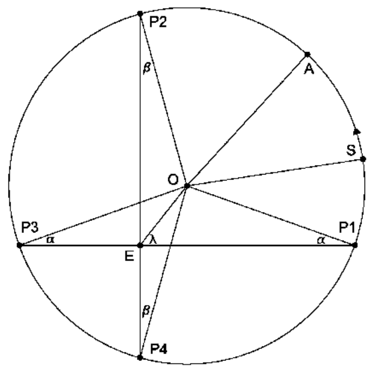

Fig. 12. Hipparchus' calibration of Apollonius eccentric model of the Sun

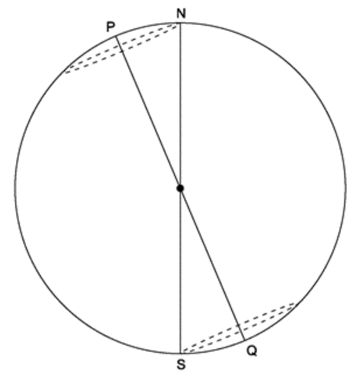

Fig. 13. The Ptolemaic representation of equinoxial precession

Another important contribution by Hipparchus was that the he measured that the time it takes for Sun to return to the same position with respect to the fixed stars (the *sidereal* year), is about 20 minutes longer than the time it takes for the Sun to return to the same solstice or equinox (the *tropical* year). This means that the points at which the celestial equators and the ecliptic plane intersect, the equinoxes, slowly move with respect to the background stars. Hipparchus measured that this *precession of the equinoxes* occurs at a rate of 1° per century—the actual value is 1° every 72 years. This means that a star that at a certain moment coincides with an equinoxial point makes a full eastward circle around the ecliptic to return to the same equinoxial point after about 26000 years. Ptolemaic astronomers coped with this phenomenon (see Kuhn 1995, 268) by adding a second sphere to explain the motion of the celestial sphere—and the same method can be used in any geocentric model (see Figure 13). The first sphere rotates with a period of a sidereal day around the celestial north-south axis. The axis of the second sphere is perpendicular to the ecliptic (inclined 23°40' with respect to the axis of the first sphere), and its endpoints are fixed on antipodal points on the first sphere. The rotation period of the second sphere is 26000 years. As a result, the position of the celestial poles slowly change with respect to the fixed stars: a star that at some instant coincides with the celestial north pole, makes an eastward circle around the ecliptic northe pole, to return the celestial north pole after 26000 years.

Hipparchus did not develop a model of deferents and epicycles for the representation of the motion of the planets. That achievement is due to Ptolemy (ca. 100 – ca. 170 AD), who in his *Mathematical Sintaxis*, commonly known as *The Almagest* (Toomer 1984), introduced the geometric model of the Universe that dominated astronomy for more than a millennium.

Ptolemy's model for the Sun was basically the same as the one introduced by Hipparchus. An important improvement was that he was able to predict the position of the Sun at any given time along its non-uniform (as seen from the eccentric Earth) path along the ecliptic. In figure 14, A is the apogee, O the center of the Sun's orbit, and S is the Sun. The angle $\bar{\alpha} = AOS$ increases uniformly. Given O (from Hipparchus model) and the tropical year, $\bar{\alpha}$ can be calculated at any given time. For the angle $\alpha = AES$, it is clear that $\alpha = \bar{\alpha} \pm \delta$. Ptolemy was able to compute the angle δ as a function of $\bar{\alpha}$, allowing thus to predict the position of the Sun at any given time, given by the angle α.

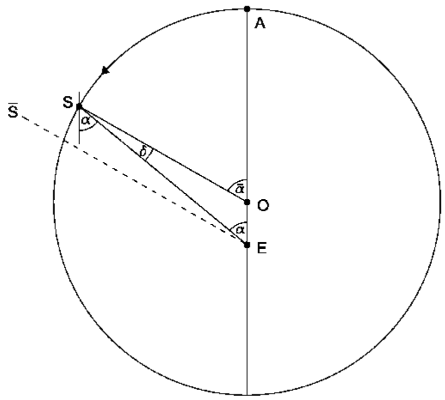

Fig. 14. Ptolemy's model of the Sun

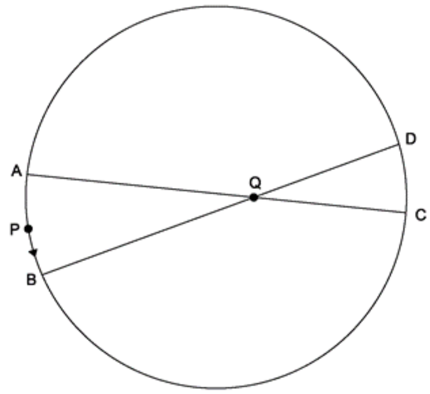

Fig. 15. Ptolemy's equant

Ptolemy also introduced a crucial concept, the mean sun $\bar{S}$, i.e., the point in the ecliptic that the Sun would occupy at a certain time if it moved with uniform angular speed as seen from the Earth. $\bar{S}$ is a crucial parameter in the calibration of Ptolemy's models for each celestial object. As it can seen in figure 14, the longitude of $\bar{S}$ at a time t as determined from E, i.e., the angle $AE\bar{S}$, is given by $\bar{\alpha}$.

Ptolemy's representation of the motion of the planets was based on Apollonius' idea of deferent and epicycle—we saw above how this method can represent retrograde motion. Now, to solve the problem of the variable angular speed of the planets along the zodiac, introducing a deferent eccentric with respect to the Earth for each planet seemed the natural strategy. However, Ptolemy noticed that this would not be enough, so he invented a novel geometric device, the *equant*—a point with respect to which the center of the epicycle moves along the deferent with constant angular velocity (see Figure 15). Since the equant is not the center of the deferent, it is clear that the motion of the center of the epicycle is not uniform along the deferent—$DQC = AQB$, so the point P covers the arcs $AB \neq CD$ in the same time. The introduction of the equant, although strongly increases the empirical adequacy of the representation of celestial motions, violates the principle of circular *uniform* motion of celestial bodies.

Figure 16 illustrates Ptolemy's model for superior planets. E is the Earth, O is the center of the deferent, and Q the equant—the three points lie on a straight line and $EO = OQ$. C is the center of the epicycle, and it rotates counterclockwise along the deferent. The angle $\bar{\lambda}$ is the *mean longitude* of the planet, and since Q is the equant, $\bar{\lambda}$ increases uniformly with time. The angle $\bar{\lambda}$ for a 'mean planet' is analogous to the $\bar{\alpha}$ for the mean Sun $\bar{S}$ in Figure 14, but this time determined from V, the vernal solstice, rather than from the apogee. Thus, $\bar{\lambda}$ is subtended by QC and a straight line from Q parallel to EV. The planet is placed at P, which moves counterclockwise and uniformly along the epicycle, so that the angle $\bar{\mu}$ increases uniformly.

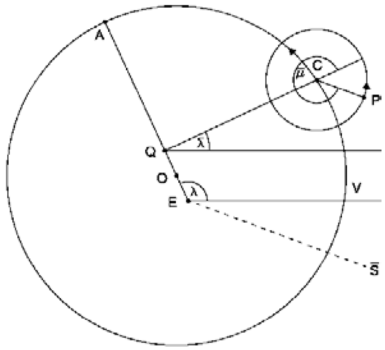

Fig. 16. Ptolemy's model of a superior planet

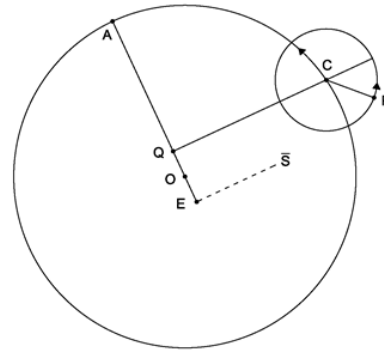

Fig. 17. Ptolemy's model of Venus

This gives us the conceptual basis of the model, but then Ptolemy had to calibrate it according to some parameters to save the observed phenomena. A first empirical constraint is that with T_z and T_s the zodiacal and synodic periods of the planet, respectively, and with T the tropical year, it holds that $\frac{1}{T_z} + \frac{1}{T_s} = \frac{1}{T}$. To satisfy this constraint, Ptolemy arranged the model as follows. The period of C around the deferent must be T_z, and the period of P around the epicycle must be T_s. Thus, at a given time t, the angles $\bar{\lambda}$ and $\bar{\mu}$ are given by $\bar{\lambda} = t/T_z$ and $\bar{\mu} = t/T_s$. It follows that the longitude of the mean sun $\bar{S}$ (measured from the solstice V) at t is then $t/T = \bar{\lambda} + \bar{\mu}$, which in turn enforces that CP, the line joining the epicycle center and the planet *is always parallel to* $E\bar{S}$, the line joining the Earth and the mean Sun—this feature will be important below. Finally, by means of rather complicated methods, Ptolemy determined the ratios OA/EQ and OA/CP for each planet, so that he could calculate the true longitude of the planet at any time, given by the angle VEP. An important subtlety in the model was that the angle $\lambda = VEA$—where A is the deferent's apogee, which is assumed to be fixed with respect to the background stars—slowly increases due to the precession of V.

Figure 17 represents Ptolemy's model for Venus. For inferior planets, the elongation with respect to the Sun is always small. To represent this, the center of the epicycle C always coincides with the real Sun, so C moves along the deferent around and the equant Q, at a rate such that QC *and* $E\bar{S}$ *are always parallel*. The remaining features are basically the same as for exterior planets. C moves counterclockwise along the deferent with center in O; E, O and Q lie on a straight line, and the eccentricity EO is equal to OQ; the planet is in P, which rotates around C counterclockwise. The model for Mercury is more complicated, but it retains these basic features—including that QC is always parallel to $E\bar{S}$.

A simplified representation of Ptolemy's model—underscoring the fact that the line joining the center of the epicycle and the position of the planet in the case of superior planets, the line joining the equant and the center of the epicycle in the case of inferior planets, and the line joining the Earth and the mean Sun, are always parallel—is presented in Figure 18. It is clear that these lines being always parallel is a contingent feature of the model. Actually, it is a rather puzzling feature: of all the possible kinematic configurations of celestial objects, why one in which the mean Sun plays this special role? The question is especially pressing if we consider that there is nothing dynamically special about the Sun in the model, let alone about the mean Sun. Thus, this particular configuration does not get any explanation, and remains literally as a cosmic coincidence in Ptolemy's model.

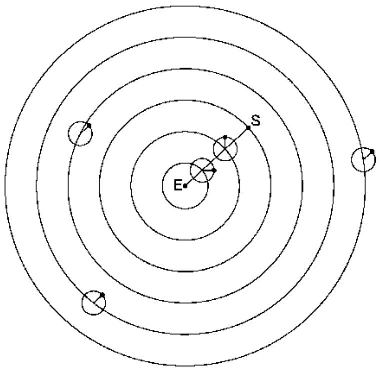

Fig. 18. Simplified sketch of Ptolemy's model

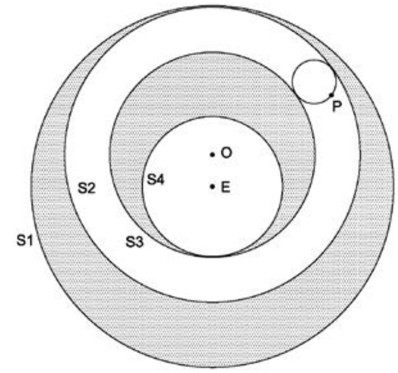

Fig. 19. Ptolemy's planetary spheres

What we have said so far accounts for the longitude of the planets. In order to represent their latitude, Ptolemy tilted the plane of the deferent with respect to the plane of the ecliptic, and also the plane of the epicycle with respect to the plane of the deferent. The values for these angles were determined by Ptolemy for each planet. As we will see below, from a heliocentric perspective the Ptolemaic epicycle of exterior planets models the motion of the Earth around and the Sun, so the plane of the epicycle should be parallel to the ecliptic. But since Ptolemy had no way to know this, he had to determine the two tilting angles separately. Furthermore, he arranged that the inclination of the deferent with respect to the ecliptic varies as a function of time. As a result, the Ptolemaic theory of planetary latitudes is rather cumbersome.

As for the physical interpretation of the model, Ptolemy does not present explicit statements about it in the *Almagest*. However, in a later work entitled *Planetary Hypotheses*, he did assume a realist view. That is, he claimed that the circles of deferents and epicycles are determined by physical ethereal spheres, arranged as represented in Figure 19. E is the Earth, the center of the universe, and O is the center of the deferent corresponding to the planet P. The 'thick ethereal sphere' of planet P is then situated in the annulus between S_2 and S_3. If we take P to be Jupiter, we can locate Mars within S_4 and determine its own 'thick sphere' in an analogous way, and the same arrangement can be set for Saturn outside S_1.

An interesting consequence of this physical interpretation is that it allows a method to determine planetary distances from the Earth. The principle is rather simple: assuming that the thick spheres are disposed as close together as possible (following a *horror vacui* principle), the maximum distance of a planet is equal to the minimum distance of the next outer planet. As we saw, Ptolemy had determined the ratios between deferents and epicycles radii for each planet, so the physical realist interpretation allows to determine relative distances for all planets and even the celestial sphere. Furthermore, in the *Almagest*, Ptolemy, using parallax and geometrical reasoning, had calculated a maximum distance for the Moon of 64 Earth radii, which corresponded then to the minimum distance of Mercury. The rest of the distances are given in Table 2.[15] If we use the *stadion*, the Greek unit that Eratosthenes (ca. 276 BC – ca. 195 BC) employed in his (roughly correct) calculation of the circumference of the Earth—from which its radius can be obtained—we get an estimation of the size of the universe resulting from Ptolemy's method. Eratosthenes obtained a circumference value of

[15] Interestingly, the distance that Ptolemy obtained for the Sun by this method in the *Planetary Hypotheses* (1079 Earth radii) is very close to the value he had obtained in the *Almagest* by a different and independent method (1160 Earth radii). For a treatment of this issue, see Carman (2010).

250.000 *stadia*. With some basic geometry, and picking a value of 160 meters for the *stadion*,[16] it follows that radius of the Ptolemaic universe was *ca.* 128.000.000 kilometers.[17]

	Minimum	Maximum
Moon	33	64
Mercury	64	166
Venus	166	1079
Sun	1079	1260
Mars	1260	8820
Jupiter	8820	14187
Saturn	14187	19865
Fixed Stars	20000	

Table 2. Distances to the Earth, in Earth radii,
as determined from Ptolemy's model

Ptolemy's model did not suffer any significant modification for centuries. Only when the Islamic medieval civilization flourished, Arabic astronomers introduced some amendments and criticisms (see Linton 2004, Ch. 4). A curious development was related to Ptolemy's measurement of the obliquity of the ecliptic with respect to the celestial equator—he got a value of 23°51'. Thabit Ibn Qurra, an Arabic astronomer of the 8th century, obtained a more accurate value of 23°33', but he took Ptolemy's measurements as correct, concluding that the obliquity of the ecliptic varies through time. Furthermore, Ibn Qurra calculated 82°45' for the longitude of the apogee (the angle λ in Figure 12). The change in value from Hipparchus and Ptolemy's times was natural given precession (recall Hipparchus obtained 65°30'), but since he trusted the different estimations of precession rates obtained by preceding astronomers, which differed from the one he calculated, he concluded that precession rate was also a variable function of time. In order to cope with the alleged varying obliquity of the ecliptic and variable rate of precession, Thabit introduced a *trepidation* theory for the ecliptic in Ptolemy's model.[18] Al-Battani, also in the 8th century, measured an ecliptic obliquity of 23°35', but he did not conclude a varying value, he simply corrected Ptolemy's result and improved the model of the Sun—amending also the mean Sun motion, the precession of the equinoxes rate, the longitude of the apogee, and the eccentricity of the deferent.

By the 12th century, substantial criticisms based on Ptolemy's model incompatibilities with Aristotelian principles were levelled by astronomers and philosophers (see Goldstein (1980) for an overview of medieval attacks against Ptolemy). As we already mentioned, the equant involves the rejection of *uniform* circular motion. Furthermore, the eccentricity of the deferents with respect to the Earth was hard to swallow from an Aristotelian standpoint—for some planets, the deferent center lies outside the orbit of the Moon. Prominent

[16] The value in meters of the Greek *stadion* is a disputed issue, but the proposed estimations range between 150 and 200 meters.
[17] As Kuhn (1995, 82) reports, the Arabic astronomer Al-Faraghi (800-870) applied the same method as Ptolemy in the *Planetary Hypotheses*. Using a value of 3250 Roman miles for the radius of the Earth, he calculated a universe radius equivalent to ca. 120.000.000 kilometers. That Ptolemy himself used this method was discovered only in 1967, when the relevant passage in the Arabic version of the *Planetary Hypotheses* was found and translated (see Carman 2010 and the references therein).
[18] The obliquity of the ecliptic is indeed variable, and it is due to gravitational perturbations of other planets. However, the real effect is much smaller than the one that Ibn Qurra deduced from the difference between his measurements and Ptolemy's.

thinkers like Averroes (1126-1198) (see Sabra 1984; Çimen 2018) and Maimonides (1138-1204) (see Nutkiewicz 1978) objected to Ptolemy's model on these grounds.

However, attempts to build models that were faithful to Aristotelian physics were unsuccessful. During the 12th century, Nur ad-Din Al-Bitruji proposed a model reminiscent of Eudoxus' idea of concentric spheres (see Goldstein 1971; Sabra 1984; Çimen 2018), and, as late as the early 16th century, the Italian astronomers Girolamo Fracastoro and Gianbattista Amico tried to revive Eudoxus' model (see Dreyer 1953, 296-304), but, all such attempts were of no avail.

Anyhow, Ibn Al-Shatir (1304-1375) was successful in introducing a model that retained most of the basic features of Ptolemy's system, but eliminating the equant (see Kennedy & Roberts 1959). Using an arrangement of epicycles on epicycles, Al-Shatir obtained planetary orbits equivalent to Ptolemy's, but without using the equant, restoring thus the uniformity of circular motion. Although Al-Shatir's model did not have a major impact, his method to eliminate the equant was used by Copernicus in his heliocentric model. Al-Shatir is also responsible for another amendment of Ptolemy's model of the Sun. The astronomer Al-Zarqali (1029-1087) had discovered that the apogee slowly moves with respect to the fixed stars, at a rate of 1° every 279 years, or 12.9" per year.[19] Al-Shatir coped with this phenomenon by embedding his solar model within another circle along which the apsidal line rotates.

2.5. Copernicus' Model

The role of Copernicus' model in the historical development of astronomy and in the constitution of modern science is not as revolutionary as the expression *Copernican revolution* suggests. In short, Copernicus' (1992) work entitled *On the Revolutions of the Heavenly Spheres*, published in 1543, more than a novel geometric astronomical model, is a translation of Ptolemy's to a heliocentric perspective. This subtle maneuver, of course, led to revolutionary science—especially when Kepler entered the stage—but evaluated on its own merits, Copernicus' contribution was hardly groundbreaking. Actually, a central motivation for the heliocentric model was rather conservative: to reintroduce the principle of circular uniform motion. As we saw, Ibn Al-Shatir had already achieved that goal, so the main innovation of Copernicus' work was a simple explanation of retrograde motion as an optical effect due to the orbital motion of the Earth. The heliocentric translation of Ptolemy's model also allowed other advantages, like a purely geometric method to determine planetary distances. Hence the appeal of Copernicus' proposal for astronomers of the time. Despite this conceptual attractive, a more nuanced evaluation is that, although it was certainly a spark for the revolutionary work of Kepler and Galileo, "Copernicus might well be described as the last of the ancients, a spiritual companion of Aristarchus, Hipparchus, and Ptolemy" (Linton 2004, 121).

Copernicus' model assigned two basic motions to the Earth. First, a rotation about an axis fixed on the celestial poles, with a period of a sidereal day. This explains the observed diurnal motion of the stars, the Sun and the planets, but now the celestial sphere is at rest. Second, the Earth undergoes a circular translational motion around a fixed Sun, with a period of a sidereal year (roughly 365 ¼ days) describing a plane inclined 23°28' with respect to its rotation axis. This implies of course that the ecliptic gets redefined as the orbital plane *of the Earth*.

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19 What Al-Zarqali discovered was the precession of the apsidal line in the *Earth's* orbit. The modern estimation of apsidal precession period for the Earth is 11.6" per year. Apsidal precession is caused by concomitant factors, including gravitational planetary perturbations. A full explanation can only be given using general relativity: the precession of Mercury's perihelion was a crucial element in Einstein's formulation of his gravitational theory.

This simple geometric scheme allows a simpler explanation of the precession of the equinoxes: if the poles N and S of the axis of terrestrial rotation describe a circle centered on the poles of the ecliptic P and Q in about 26.000 years, as represented in Figure 20, a star that at an instant coincides with an equinox makes a circle around the ecliptic, advancing 1° every 72 years.

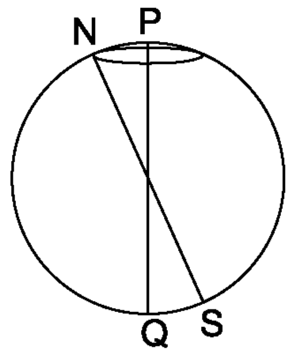

Fig. 20. Precession of the equinoxes, heliocentric perspective

Copernicus measured a roughly correct period of precession of about 26.000 years. However, his actual account of this phenomenon was rather complicated. Since he thought that the Earth is carried around the Sun by a sphere (here we find yet another conservative principle), he concluded that the axis of rotation should change direction, unless a third compensating motion, that Copernicus named *motion of the inclination*, is added to rotation and translation. Now, this third motion is not exactly opposite to the axial direction change induced by the sphere that carries the Earth, so it was the difference between both motions that accounted for precession. Besides, just like Thabit Ibn Qurra, Copernicus mistakenly believed that the rate of precession is variable in time, and that so is the obliquity of the ecliptic. Thus, Copernicus included a trepidation theory. The Copernican account of the Earth's motion was complicated by yet another factor: the model also copes with apsidal precession (discovered by Al-Zarqali), that is, the apsidal line joining aphelion and perihelion (the points at which the Earth is farthest and closest from the Sun) has a variable direction with respect to the fixed stars.

Figure 21 depicts Copernicus' model of the Earth's orbit. It moves uniformly and counterclockwise along the circle centered in O with a period of a sidereal year. In turn, O moves uniformly and clockwise along a circle centered in C, with a period of 3.434 years, and C moves uniformly and counterclockwise along a circle centered in the Sun S, with a period of 53.000 years. This complicated arrangement is connected to trepidation, and also to apsidal precession. For example, it can be seen that as a result of the motion of O around C, the eccentricity of the Earth's orbit varies, from which it follows that the aphelion A oscillates around the mean aphelion $\bar{A}$ with a period of 3.434 years. In turn, due to the motion of C around S, $\bar{A}$ describes a full circle around S in 53.000 years. This geometric arrangement guarantees that the point O, the center of the terrestrial orbit, corresponds to (the heliocentric counterpart of) the mean Sun $\bar{S}$.

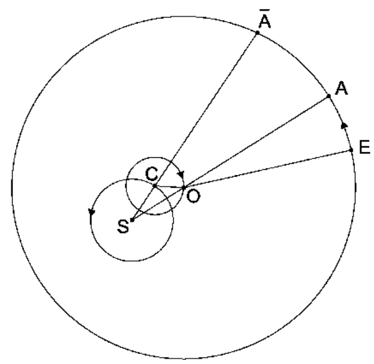

Fig. 21. Copernicus' model of the Earth

As we said, a central motivation for Copernicus's model was a simple explanation of retrograde motion, illustrated in Figures 22 and 23. The position of the planet against the backdrop of the fixed stars is given by the direction of a line from the Earth E passing through the planet P. Retrograde motion is then nothing but an optical effect due to the orbits of E and P around the Sun. Figure 21 also shows why for superior (exterior) planets retrograde motion occurs always in opposition, and figure 22 shows why for inferior (interior) planets it always occurs in conjunction.

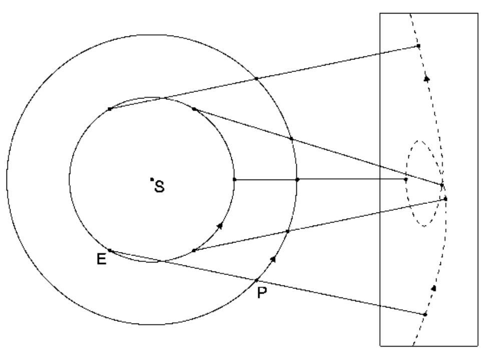

Fig. 22. Retrograde motion, exterior planet

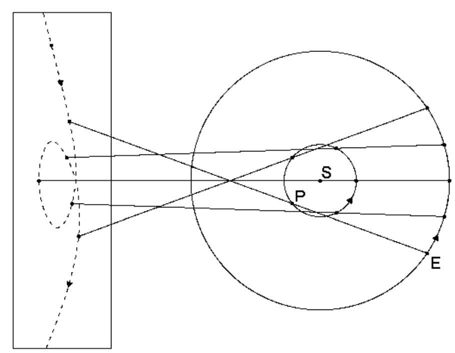

Fig. 23. Retrograde motion, interior planet

A challenge imposed by planetary motion was that their speed is variable along their path through the zodiac. To deal with it, Ptolemy invented the equant, and added it to the eccentric deferent mechanism. Copernicus was more conservative than Ptolemy in this point, so in the name of the principle of circular uniform motion, he avoided the equant and coped with this irregularity by using a deferent-epicycle system. Figure 24 displays Copernicus' model for an exterior planet. O, which is not the Sun, but lies nearby it, is the center of the deferent. The center of the epicycle C moves along the deferent with a period equal to the sidereal period of the planet. The planet at P rotates around C, in the same direction and with the same period. The motion is arranged in such a way that the angles PCO and AOC are equal, where A is the orbit's aphelion. Notice that the deferent-epicycle construction plays a different role than in Ptolemy's planetary models—now it takes care of the observed variable orbital speed of the planet, not of its retrograde motion.

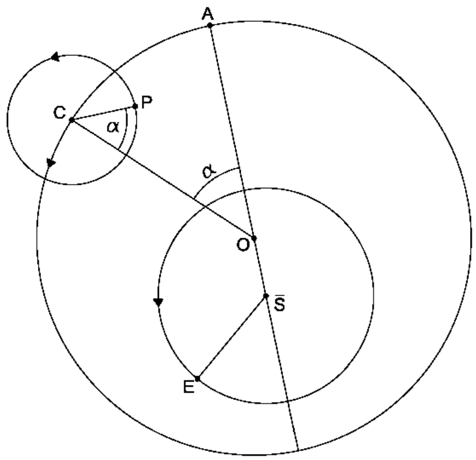

Fig. 24. Copernicus' model of an exterior planet

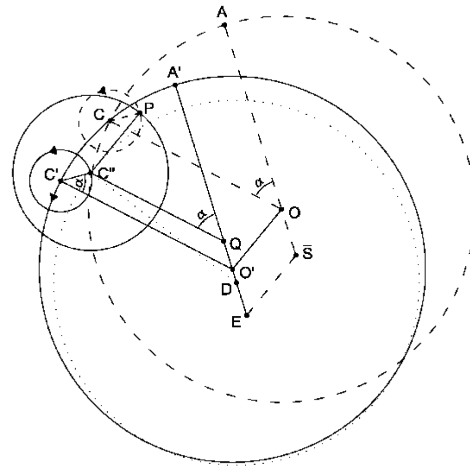

Fig. 25. Geometric inter-translability between Copernicus' model for an exterior planet, Ptolemy's, and a geocentric model without

The geometric translation to a geocentric standpoint is illustrated in Figure 25. In Copernicus' model (dashed lines), the Earth E rotates in a circular orbit centered in the mean Sun $\bar{S}$. The vector that gives the position of the planet as seen from the Earth is $E\bar{S} + \bar{S}O + OC + CP$. The figure superimposes Copernicus' model with a geocentric model (solid lines) with two epicycles. O' is the center of the deferent with radius $O'C'$, C' is the center of epicycle with radius $C'C''$, and C'' is the center of the epicycle with radius $C''P$. The position vector of the planet is now $EO' + O'C' + C'C'' + C''P$. It is clear that EO' is parallel to $\bar{S}O$, and that $|EO'| = |\bar{S}O|$. The same hold for $O'C'$ and OC, for $C'C''$ and CP, and for $C''P$ and $E\bar{S}$. It follows then that the planet position vector in the heliocentric model is the same as the planet position vector in the geocentric model.

To show the translatability to Ptolemy's model, we draw the line $C''Q$, parallel to $O'C'$, so that the angles AOC and $A'QC''$ are equal and increase uniformly. Now, $ED = DQ$, so the center of Ptolemy's deferent must be D. If it holds that $|\bar{S}O| = 3|CP|$, then $|OC|$, the radius of Copernicus' deferent, and $|DC''|$ are equal (see Linton 2004, 140-141). Copernicus' devised his model so that this condition is met, so Q is the equant of the Ptolemaic deferent (dotted line) with radius DC'', and the radius of the Ptolemaic epicycle is then $C''P$, so C'' rotates along the Ptolemaic deferent according to the equant rule. Then, in the Ptolemaic model the planet position vector is $ED + DC'' + C''P = EQ + QC'' + C''P$, and since $EQ + QC'' = EO' + O'C' + C'C''$, all three planet position vectors are the same. We have illustrated then how the equant can be dispensed with in a geocentric model by using epicycles on epicycles, as Al-Shatir did (solid lines), and how this can be translated into a heliocentric model, as Copernicus did.

Our exposition of the geometric translation between Copernicus' and Ptolemy's model for an exterior/superior planet allows us to see that since the position vector in the heliocentric case is $E\bar{S} + \bar{S}O + OC + CP$, whereas in Ptolemy's model it is $ED + DC'' + C''P$, then in the geocentric case it must be the case that the line joining the Earth and the mean Sun and the line joining the epicycles center and the planet, that is, $E\bar{S}$ and $C''P$, must be always parallel. As we saw, in Ptolemy's model this is a cosmic coincidence, but considering it from the point of view of the inter-tranlsability with Copernicus' model, it gets explained away.

Figure 26 shows Copernicus model for Venus. The Earth rotates on a circle centered in $\bar{S}$, C rotates on a deferent centered in O with a period given by Venus' sidereal period, whereas the planet rotates along the epicycle centered in C at twice the rate that the Earth rotates around $\bar{S}$, so that the angle $CO\bar{S}$ is twice the angle $E\bar{S}A$, where A is the point at which the Earth is farthest from O.

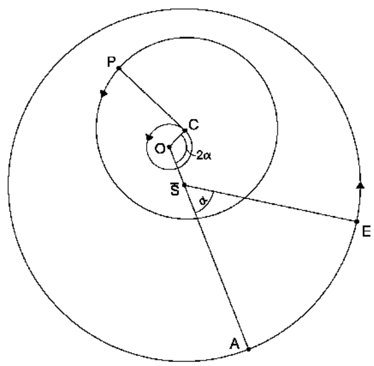

Fig. 26. Copernicus' model of Venus

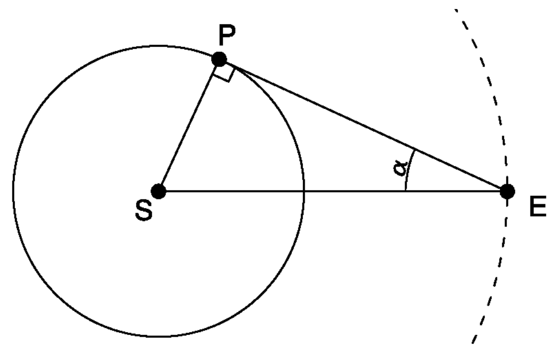

Fig. 27. The distance of an interior planet to the Sun in Copernicus' model

The heliocentric configuration of interior planets also explains away the coincidental features of Ptolemy's model. Since an inferior planet in the geocentric model is an interior planet in the Copernicus' model, then it must be the case that its maximum elongation is a small angle (Fig. 27)—this is just a contingent feature in Ptolemy's model, but if we take it as the geocentric translation of Copernicus' model, then it must be arranged in this way.

Furthermore, the apparent motion of a planet depends on two factors: the motion of the planet around the Sun, and the motion of the Earth around the Sun. In the Ptolemaic models for the superior planets, the first factor is taken care of by the deferent, and the second factor by the epicycle. This remark offers an intuitive explanation of the Ptolemaic coincidence that for superior planets the line joining the center of the epicycle and the planet must be always parallel to the line joining the Earth and the mean Sun (cf. Fig. 25). In the case of the Ptolemaic models for inferior planets, the roles of deferent and epicycle get inverted: the deferent represents the second factor, and the epicycle represents the first one. As a result, the line joining the equant and the center of the epicycle must be always parallel to the line joining the Earth and the mean Sun (cf. Figs. 16 and 17). That is, this twofold coincidence in Ptolemy's model is explained away in the heliocentric translation.[20]

Another source of appeal of the Copernican model is a purely geometric method to estimate planetary distances to the Sun. The method is very simple for interior planets. When the planet is at maximum elongation, the Earth, the Sun and the planet form a triangle rectangle in the planet. Taking the Earth-Sun distance as our unit of length, it is clear that the planet-Sun distance PS is $\sin\alpha$ (see Figure 27). The distance to the Sun for exterior planets can also be obtained by a simple trigonometric reasoning (see Jacobsen 1999, 125). Copernicus' trigonometric method is better than the one formulated by Ptolemy. It can be easily seen that it does not rely on physical assumptions like spheres arrangement.

Furthermore, in the heliocentric model the order of the planets can be established in a more systematic way. In Ptolemy's model, the period used to organize planetary order was the zodiacal period (the average

[20] For an illustration and a rigorous treatment of the inter-translability between the Ptolemaic and Copernican models of Venus, see Neugebauer (1986, 497) and Barbour (2001, 239).

time it takes a planet to return to the same position respect to the background stars as seen from the Earth). But since the zodiacal periods of Mercury and Venus are equal to the sidereal year of the Sun, they could not be invoked to univocally settle the order for these three bodies. In Copernicus' model, the relevant period to determine planetary order is the sidereal one (the period it takes the planet to complete one orbit around the Sun). An intuitive comparison of the geometry of the models shows that for an exterior/superior planet its sidereal and zodiacal periods are the same (cf. Kuhn 1995, 167), but that is not the case for interior planets. However, the orbital periods of interior planets in Copernicus' model can be easily calculated. After the Copernican explanation of retrograde motion, for an interior planet its synodic period T_p gives the time between inferior conjunctions with the Earth (cf. Fig 23 and fn. 13). Measuring time in years, during T_p years the Earth obviously makes T_p terrestrial orbits, and the interior planet makes $1 + T_p$ revolutions around the Sun in that same time. Therefore, in one year an interior planet makes $1 + 1/T_p$ revolutions around the Sun. Thus, an interior planet makes one revolution around the Sun in $1/(1 + 1/T_p)$ years. As it can be seen from Table 1, for Mercury, $T_p = 0{,}315$ years, so its sidereal period is $87{,}5$ days; whereas for Venus, $T_p = 1{,}599$ years, so its sidereal period is $224, 8$ days. Sidereal period can be then applied to determine the order of all planets.

On the negative side, although Copernicus' model is *conceptually* simpler—recall the explanation of retrograde motion, precession of the equinoxes, and the cosmic coincidences of Ptolemy's model—in geometric technical terms the model is at least as complicated as Ptolemy's. For example, a complex use of deferents and epicycles is still present, although for purposes different than in Ptolemy's model. This is hardly surprising when we consider that Copernicus' model is, bottom line, a heliocentric geometric translation of Ptolemy's. The Polish astronomer did not introduce novel mathematical methods that could simplify the representation of the phenomena.

This can be seen quite clearly in the latitude theory. Copernicus' aim was to reproduce Ptolemy's results in this matter, so he did not realize the simple representation of planetary latitudes that a heliocentric model allows—as we will see below, Kepler did. In the Copernican model, the inclination of the orbital planes that account for observed planetary latitudes are defined with respect to the mean Sun rather than to the real Sun, and the inclinations are variable in time (as in Ptolemy's model) and connected in a highly complicated way to the longitude of the Earth. Copernicus' latitudinal theory is at least as complicated as Ptolemy's.

A second shortcoming was that an Earth in motion implies that a stellar parallax angle is predicted. As the Earth moves along its orbit, the direction of the apparent position of a star should change night after day at different locations of the Earth along its orbit, but no such variation was observed. In Figure 28, E_1 and E_2 represent two positions of the Earth along its orbit, separated by an interval of six months, the direction of the apparent position of a star T should change from E_1 to E_2, according to a parallax angle α. This means either that the thesis of an Earth in motion is wrong, or that the distance to the fixed stars is too large for parallax to be detectable—the value of α is inversely proportional to the distance ST. This was especially pressing since the method to determine planetary distance in Copernicus' model gives a solar system smaller than the one determined from Ptolemy's method in the *Planetary Hypotheses*. Using astronomical units—the Earth-Sun distance—in the geocentric model the mean distance to Saturn is 14, whereas the (correct) value obtained from Copernicus' method is 9,6. For the angle of parallax to be undetectable, the radius of the celestial sphere should be of at least a million Earth radii. Recall that the radius of the celestial sphere calculated by Ptolemy was ca. 20.000 Earth radii. A gap of such an unimaginable size between Saturn's orbit and the celestial sphere, devoid of any objects, was very difficult to accept given the ruling aesthetic and metaphysical principles of the time.

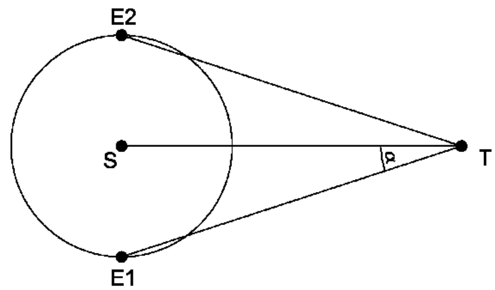

Fig. 28. Stellar parallax in a heliocentric model

Finally, although Copernicus' managed to get rid of the equant and reinstall the principle of regular circular motion, the heliocentric model is in obvious conflict with the then prevalent physics of a fixed Earth. Copernicus argued that given its spherical shape, the natural motion of the Earth, shared by all terrestrial objects, is a circle. To the modern reader this may sound like a small step towards a concept of inertia, but taken at face value is only a qualitative speculation far from being able to allow a satisfactory explanation of terrestrial phenomena for astronomers of the day. Thus, the immediate reaction to Copernicus' model was rather cautious. Although its virtues were valued, it was usually read through instrumentalist glasses.

2.6. Tycho's Model

In the second volume of his *Introduction to New Astronomy*, published in 1588, Tycho Brahe presented a geocentric model that grasps the virtues of Copernicus' model over Ptolemy's.[21] The basic idea is simple: the Earth is the center of the universe, the Sun describes a circular orbit around the Earth, and the rest of the planets describe circular orbits around the Sun. In other words, the Sun moves along a deferent, and each planet moves along an epicycle centered in the Sun. The disparity between superior/exterior and inferior/interior and planets (maximum elongation angles, retrograde motion at opposition or conjunction) is dealt with in the Tychonic model by the fact that the radii of the orbits of Mercury and Venus around the Sun are smaller than the radius of the orbit of the Sun around the Earth, as it can be seen in Figure 29.[22]

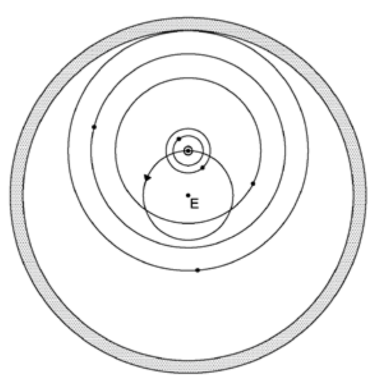

Fig. 29. The Tychonic model

[21] For a comprehensive treatment of Tycho's work, see (Dreyer 2014)
[22] The figure is a simplification. The actual model includes eccentricities and equants.

Tycho's model can be also put in geometric translation with Ptolemy's and Copernicus', as it can be seen in Figure 30 (eccentricities, Ptolemy's equant and Copernican epicycles are ignored). In Tycho's model, the Earth E is fixed, the Sun S rotates around it along the dotted circle, and a generic exterior planet P rotates around S along the larger dashed circle. In the Copernican system, S is fixed, E rotates around S along the smaller dashed circle, and P rotates around S along the larger dashed circle. In Ptolemy's model, E is fixed, S rotates around E along the dotted circle, C rotates along a deferent of radius EC, the larger solid circle, and P rotates along the epicycle centered in C. That the observed trajectory of P is the same in all three models is guaranteed by the fact that $ESPC$ is always a parallelogram.

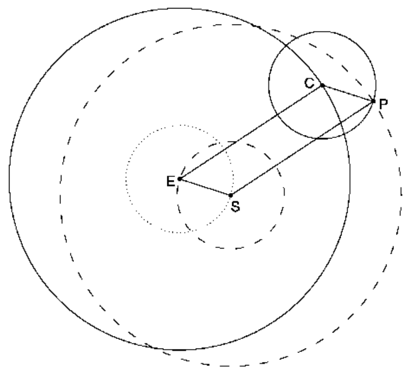

Fig. 30. The inter-translability between
Ptolemy's, Copernicus and Tycho's model

A quick inspection of Figure 29 shows that Copernicus' trigonometric method to determine planetary distances can be run in the Tychonic model too. On the other hand, as in Ptolemy's model and unlike Copernicus', retrograde motion is explained in terms of the deferent-epicycle method. However, Tycho's model manages to explain why for inferior planets it occurs at conjunction, whereas for superior planets it occurs at opposition (cf. Fig 11). Furthermore, recall that in Ptolemy's system the effect of the motion of the Earth around the Sun is taken care by the epicycle for superior planets, and by the deferent for inferior planets. In Tycho's model it is the deferent that plays that role for all planets, and since the Sun in the center of the deferent of each planet, there is no need to fine-tune the model with respect to the (mean) Sun.[23]

The Tychonic model also avoids the two most important shortcomings of Copernicus'. Since it is geocentric, the problems for terrestrial physics related to an Earth in motion do not come up. Besides, no stellar parallax is expected, so Tycho could simply assume that the sphere of the fixed stars lies just after Saturn's orbit. As a result, the size of the universe determined in Tycho's model is smaller than the Ptolemaic universe.

After this outline of the Tychonic model, and in evaluative comparison with Copernicus' and Ptolemy's models, it is clear that for contemporary astronomers it was the best of both worlds. As Kuhn reports:

> The remarkable and historically significant feature of the Tychonic system is its adequacy as a compromise solution of the problems raised by the *De Revolutionibus*. Since the Earth is stationary and at the center, all the main arguments against Copernicus' proposal vanish. Scripture, the laws of motion, and the absence of stellar parallax, all are reconciled by Brahe's proposal, and this reconciliation is effected without sacrificing any of Copernicus' major mathematical harmonies. (Kuhn 1957, 202)

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[23] For the precession of equinoxes, Tycho's system must go back to an explanation like the one illustrated in Fig. 13.

2.7. Kepler's Vicarious Hypothesis

Tycho Brahe is a very important character in our story also for his meticulous and abundant observations of the behavior of celestial objects. He was able to obtain observational data for the positions of planets, at different times and in various kinematic configurations, with a margin of error of an angle of 2', whereas the margin of error the observations that Ptolemy used was of about 10'. Johannes Kepler (1571-1630) worked as Brahe's assistant for a short period, and after Brahe's death in 1601, he inherited the invaluable collection of observational data. Armed with it, he introduced subtle but crucial amendments in Copernicus' model. However, the result was a demonstration that at best, the Copernican model would lead to an observational error of 8', and given the geometric inter-translation between the models, the same holds for Ptolemy's and Brahe's models.

In his first major work, the *Cosmographic Secret*, published in 1597, Kepler (1981) defended Copernicus' model. This book is mostly known for the thesis that the order and distances of the planets can be represented using the five platonic regular solids, following a Pythagorean aesthetic-metaphysical spirit. This is normally presented as an example of Kepler's extravagant commitment to ancient and conservative views. However, although he never gave up this Pythagorean spirit, in that same work Kepler took a stance towards the Copernican model that is one of the catalysts for the scientific revolution. He noticed that, comparing planets from closest to farthest to the Sun, the orbital periods increase at a rate greater than the rate at which the distance increases. This means that the farther the planet, the slower its speed. Kepler concluded that a force emanated from the Sun, which gets weaker with distance, governs all planetary orbits. That is, he arrived to the view that a dynamical mechanism underlies the Copernican model. Before Kepler, nobody had understood an astronomical model in those terms. The Copernican model was either interpreted in instrumentalist terms, or in realist teleological terms. That is, either the model was taken only as a mathematic configuration of the observed phenomena; or as a true description in which the circular motions that determine the orbits are an expression of natural motion, not the outcome of a force. Furthermore, Kepler realized that his dynamical-mechanical thesis implied that the speed of a planet along its orbit is variable, for its distance to the Sun is variable. Thus, he openly rejected the dogma of circular uniform motion for celestial objects.

Kepler (1992) developed this view in his *New Astronomy*, published in 1609. He arranged Copernicus' heliocentric model according to the following principles. First, all planetary orbits are given by a single circle—he fully abandoned the deferent-epicycle method. Second, all orbital planes intersect in the real Sun. Third, there is an equant for each orbit, collinear with the Sun and the orbit's center, and such that the center of the orbit lies between the Sun and the equant—from a heliocentric point of view, the equant corresponds to the mean Sun, of course.[24] The last two principles capture Kepler's mechanical-dynamical stance, for they clearly suggest that the Sun is the driving force determining the orbits, and that the force decreases with distance. Given the collinear arrangement of the equant, orbital center, and the Sun, the velocity of the planet is maximum and minimum at perihelion and aphelion, respectively. As it is clear, this is much simpler than Copernicus' original plan. Epicycles are abandoned, and the latitude of a planet as observed from the Earth is simply a function of the inclination between the planet's orbital plane and the ecliptic.

Given this amended heliocentric configuration, Kepler set himself to the task of determining where exactly the orbit center lies between the equant and the Sun, starting with the most challenging case, Mars. He approached this task using both Mars' longitude and latitude. In Figure 31, A and P are Mars' aphelion and

[24] Copernicus' made extensive use of the mean Sun, as we saw above, but never as an equant.

perihelion, respectively, S is the Sun, O the orbit center and Q is the equant. The angle $\bar{\alpha}$, which increases uniformly and can be obtained from the orbital period, is the mean anomaly, and α the true anomaly. The latter angle can be expressed as a function of the former by the formula $\alpha = \bar{\alpha} - (f + g)\sin\bar{\alpha} + \frac{1}{2}g(f + g)\sin 2\bar{\alpha}$, which matches Brahe's data if $f/g \approx 0{,}64$.[25]

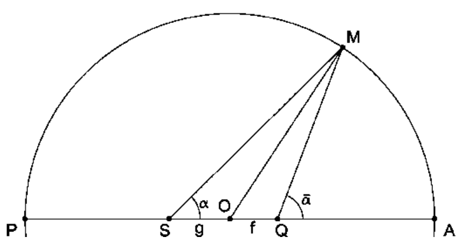

Fig. 31. Kepler's method to calculate the ratio f/g from Mar's longitude

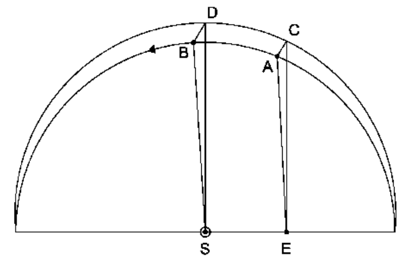

Fig. 32. Kepler's method to calculate the inclination between the ecliptic and Mars' orbital plane

Figure 32 displays one of the methods by which Kepler determined the inclination of Mars' orbital plane with respect to the ecliptic. E is the Earth, S is the Sun, and C and D lie on the ecliptic plane. Using again Brahe's data, Kepler extrapolated the position of Mars at A, that is, as seen from the Earth in a configuration such that E lies on the line of the nodes of Mars orbit (the points in which Mars orbital plane intersects the ecliptic plane), so that CAE is a right angle. In this configuration, the angle CEA is equal to DSB, where B is one of the *limits* in Mars' orbit—the points in which Mars' ecliptic latitude is maximum and minimum. It is clear that $CEA = DSB$ is the angle of inclination of Mars orbit, for which Kepler got a value of 1°50'.[26]

With this result, Kepler was able to determine the ratio f/g once again, for its value has an impact on the predicted maximum latitude of a planet as seen from the Earth. For Mars, the latitude as seen from the Earth is maximum when the planet is in opposition, as in Figure 33. The angle β is determined observationally, α is known to be 1°50', the center of Mars orbit, the equant Q and the distance SE are known, so it is clear that f/g can be obtained. The problem was that Kepler found a value of 1 this time. But with this value, the model fails for planetary longitude. When $\bar{\alpha} = \pm 45°$ (cf. Figure 31), the predicted and observed positions of Mars differ by 8'—before Brahe's data, this error would have been negligible, but with an accuracy of 2', it was unacceptable.

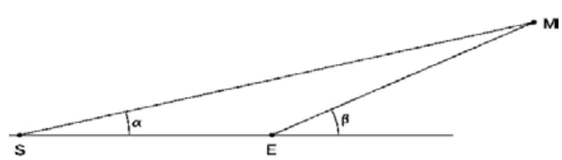

Fig. 33. Kepler's method to calculate the ratio f/g from Mar's latitude

[25] This formula for α is obtained using mathematical techniques that were not available to Kepler. The calculations he had to make are tortuous and tedious. Furthermore, the data he had to obtain the angle $\bar{\alpha}$ were observations for M in kinematic configurations with respect to the *mean* Sun, rather than to the real Sun, so Kepler had to extrapolate them. (see Linton 2004, 179).

[26] A remarkably accurate result, for the true value of the inclination of Mars' orbit is 1°51'. Kepler determined the nodes of the orbit of Mars from Brahe's data. He found that the red planet returns to the same node every 687 days, which is exactly its sidereal period, and that the Sun lies on the line joining both nodes. This is evidence for Kepler's assumption that the Sun lies on the orbital plane of each planet.

Kepler considered the possibility that the discrepancy could be rooted in an imprecise knowledge of the Earth circular orbit—as it is clear from Figure 33, the calculation of f/g crucially depends on an accurate knowledge of the position of E. Kepler devised an ingenious method to determine the circle of the Earth's orbit, shown in Figure 34. He used observations of Mars every 687 days, so that it is the same position with respect to the Sun S and the distant stars—Kepler chose moments when Mars crosses the ecliptic, to avoid complications concerning latitude. Since the orbital period of the Earth is different than Mars', the former will be located at different positions at each observation. With three positions, a unique orbital circle can be determined. With a fourth position it can be checked if it lies in the same circle. The center C of the found orbit can thus be established, and also its distance to the equant. The solid circle is Kepler's improved terrestrial orbit, whereas the dashed circle is Copernicus'. The result confirmed Kepler's mechanical-dynamical interpretation of the Copernican model: the Sun, the orbital center and the mean Sun are collinear, with the orbital center between the Sun and the equant, so that the Earth's speed is fastest at perihelion and slowest at aphelion. However, for f/g in the terrestrial orbit he also obtained a value 1, so he concluded that the same should hold for Mars, and that the observational error of 8' could not be corrected.

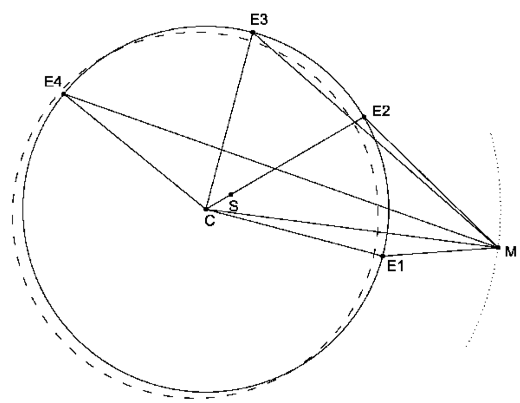

Fig. 34. Kepler's method to determine the Earth's orbit

Kepler was fully aware of the inter-translability of the models, so the results of his amendments to Copernicus' model—that made it as empirically adequate as possible, but that also showed that it is false—can be introduced, *mutatis mutandis*, in Ptolemy's and Brahe's models as well. Therefore, a choice between the three models would have to be done in terms of extra-empirical considerations. As Barbour states, "by giving the demonstration in all three systems, Kepler highlighted their equivalence at the kinematic level and emphasized that the choice between the rival systems must be assessed primarily on physical and dynamical arguments" (Barbour 2001, 294).

Kepler demonstrated thus that the three models are equally false. However, he continued using his amended version of the Copernican system as a *vicarious hypothesis*, as he called it, i.e., as a false surrogate model that would lead him to correct principles. The strategy certainly paid off. He derived the second and first laws working on the vicarious hypothesis, which resulted in a new heliocentric model with elliptical orbits.[27] Kepler's vicarious hypothesis is then the last episode in circular motion astronomy.

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[27] Kepler's first law states that the planets describe elliptical orbits, with the Sun in one of the *foci*. His second law tells us that a straight line from the planet to the Sun sweeps out equal areas in equal times. The first two laws were formulated in the *New Astronomy*. The third law states that the ratio D^3/T^2 has the same value for all planets, where D is the

Kepler took issue with the choice between the three models. He first considered Ptolemy. We know that in Ptolemy's system, for superior planets the role of the epicycle is to take care of the effect of the motion of the Earth around the (mean) Sun. After his correction of the orbit of the Earth, the translation to the Ptolemaic model should be such that the same geometric arrangement must be mirrored by the Ptolemaic epicycle. That is, in the amended Ptolemaic model for superior planets, there should now be an equant for the epicycle, collinear with a *punctum affixionis* (corresponding to the real Sun), and the epicyclic center lies exactly midway between these points. This means that in Ptolemy's system the models of the three superior planets and the model of the Sun must all be constructed on the basis of the terrestrial motion in Kepler's amended version of the Copernican model. Thus, Kepler concluded that "when a comparison of hypotheses has been made, and it has appeared that four theories of the sun [...] can be generated from a single theory of the earth, like many images from one substantial face, the sun itself, the clearest of truth, will melt all this Ptolemaic apparatus like butter, and will disperse the followers of Ptolemy, some to Copernicus' camp, and some to Brahe's" (Kepler 1992, 337).

This argument does not hold in the Tychonic model. However, in Kepler's vicarious model, leaving the Earth aside for a moment, the apsidal lines of the five ancient planets all converge in a single point: the real Sun. Furthermore, for the five planets it also holds that the orbital speed is minimum at aphelion and maximum at perihelion. In the translation to Tycho's model, both these feature must hold too. However, if the Sun moves around the Earth, the speed of the former in its orbit is fastest at apogee and slowest at perigee. Kepler realizes that a much simpler and unified arrangement obtains in the vicarious model: the apsidal line of the terrestrial orbit meets all the other planetary apsidal lines in the real Sun too, and its orbital speed is also maximum and minimum at perihelion and aphelion, respectively.

This general pattern in the vicarious is of course quite coherent with Kepler's mechanical picture of the solar system, and his discovery of the second law provides further support for it, so let us briefly see how the postulation of a solar force helped Kepler to find it. He first derived that at perihelion and aphelion, the linear velocity of the planet is inversely proportional to the distance to the Sun. A reconstruction of Kepler's reasoning, in modern terms, is offered by Barbour (2001, 303). In Figure 35, A is the aphelion, P the perihelion, C the orbit center, S is the Sun and Q is the equant, and $SC = CQ$. By the definition of the equant, in a time t the planet sweeps an angle wt, where w is the (constant) angular speed. Thus, the linear velocity of the planet at A is $v_A = w \cdot EA$, and at P it is $v_P = w \cdot EP$. Now, since $EA = SP$ and $EP = SA$, $v_A / v_P = SP/SA$, which in turn implies that both at A and P the linear velocity of the planet is inversely proportional to the distance of the planet to the Sun. The inverse proportionality holds strictly only at perihelion at aphelion, but in agreement with his solar force hypothesis, Kepler postulated it as correct for the whole orbit, so the orbit with an equant became a good approximation.

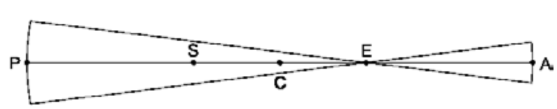

Fig. 35. At the apsides, the velocity of the planet is inversely proportional to the distance to the Sun

planet's average distance to the Sun, and T is its orbital period. Kepler obtained his third law in *The Harmony of the Universe*, published in 1619.

From a modern perspective, Kepler's "inverse distance law" can be expressed as $\frac{d\theta}{dt} \propto \frac{1}{r}$, where θ is the angle covered by the planet (as determined from the center of the orbit) as a function of time, and r is the distance to the Sun. Thus, to predict the orbit of a planet according to this law, an integration problem must be solved, for $t \propto \int r d\theta$. Kepler could not use calculus, of course, so he devised the following method. He divided the orbital circle in 360 equal arcs of length $\pi R/180$, where R is the orbital radius. The distance to the eccentric Sun in each arc is variable, but taking the average distance as the constant distance r of each arc, then the velocity of the planet along each arc is inversely proportional to r. This method was rather tedious, for in order to calculate the position of a planet at a certain time t, it was necessary to calculate and add the traversed distances in all the previous arcs up to t.

To simplify the task, Kepler reasoned rather mysteriously that "since I knew that the points of the eccentric are infinite, and their distances are infinite, it struck me that all these distances are contained in the plane of the eccentric" (1992, 418). He applied the same principle to arcs of the eccentric circle, that is, the area enclosed by an arc and straight lines from its endpoints to the eccentric Sun 'contains' all the infinitely many different distances from the points in the arc to the Sun. This slapdash reasoning gave him the clue that by calculating that area, a much easier task, he could obtain the velocity and time of the planet along the corresponding arc, according to the inverse distance law. That is, the inverse distance law could be formulated as an *area law* that the velocity of the planet along the arc is inversely proportional (and thus the time directly proportional) to the corresponding area, *if the area is proportional to the distance to the Sun*. This condition holds for triangles whose base is an infinitesimal arc and whose height is the distance to the Sun, but the distance to the Sun is perpendicular to the arc-base only at perihelion and aphelion. However, with small eccentricity, the area law is a good approximation of the inverse distance law. That is, Kepler obtained his revolutionary second law that a straight line from the planet to the Sun sweeps out equal areas in equal times using the vicarious model of circular orbits, and as an approximation of the inverse distance law, which he formulated on the basis of his dynamical assumption that the Sun governs planetary orbits by means of a force.

3. UNDERSTANDING AND EXPLANATION IN CIRCULAR-MOTION ASTRONOMY

After our review of the rise and fall of circular-motion astronomy from Eudoxus to Kepler, we can use this historical episode as a case-study to evaluate the pragmatic account of understanding proposed by de Regt. The role of scientific intelligibility (UT), the variability of standards of intelligibility, the relevance of metaphysics for scientific understanding, and the possibility of getting understanding through false models, are all features that can be recognized in the development of astronomy from Eudoxus to Kepler.

3.1. CIRCULAR (UNIFORM) AND SCIENTIFIC INTELLIGIBILITY

The ancient Greek view that the motion of celestial bodies is circular and uniform was an aesthetic-metaphysical principle that operated as the theoretical basis for the development of astronomy up to the times of Kepler. As we saw, an important point to underscore about this principle is that it is essentially teleological. The idea that nature behaves the way it does because there is a goal given by the essence of all natural things was developed by Aristotle, but it can certainly found in earlier thinkers like Plato and some pre-Socratic philosophers. The aesthetic perfection that the ancient Greeks attributed to circles was naturally associated with the stars and planets, given the degree of uniformity observed in their motion. Thus, the stars and the planets were considered to move uniformly in circles not due to mechanical factors, but simply because that is their *télos*. Barbour characterizes Greek astronomy as *motionic* (as opposed to dynamic):

> the fundamental law of ancient Greek astronomy stated that all celestial bodies move in perfect circles at a uniform (perfectly constant) speed. In accordance with this law, the motion as such *is entirely independent of all the other bodies and matter in the universe.* (Barbour 2001, 52; my emphasis)

This law of Greek astronomy, if it is to provide any scientific explanation of astronomical phenomena, and not only a qualitative-metaphysical account, must be made intelligible, in the sense of de Regt's UT. That is, from the law of circular uniform motion, models that obey the mentioned law must be built to represent the target phenomena—the motion of celestial objects. Simplicius' anecdote that Plato assigned the task of building a geometrical model that obeys the principle of circular uniform celestial motion, can be certainly understood from this point of view. For the ancient Greek thinkers, something very substantial would be gained the understanding of nature if the metaphysical principle of circular uniform model got incarnated in what we nowadays call a scientific model.

Eudoxus' method of concentric spheres was the first toolkit of intelligibility with the use of which a scientific model could be built out of the law of ancient astronomy. The method works as the mediator between the theoretical principle of circular uniform motion and the observed phenomena. It is also clear that the task of formulating models out of this basic law was not algorithmic. The idea that the combined motion of concentric spheres can reproduce regular observed motions which are not circular is due to Eudoxus' ingenuity, and to his knowledge of geometry. Thus, making the principle of circular uniform motion scientifically intelligible required skills on Eudoxus' part, which in turn depended on subjective and contextual factors. The success or failure of the model, though, depended on its empirical adequacy, and it soon became clear that the system of concentric spheres is not able to represent the phenomena with acceptable accuracy.

Apollonius introduced a second toolkit of scientific intelligibility for the same law of circular motion. Now, in order to build accurate models following his basic idea of deferents and epicycles, observational and mathematical problems needed to be solved, and their solution also required ingenuity, and also a technical refinement of the basic toolkit was needed. The development of astronomy from Hipparchus to Copernicus can actually be understood as an improvement of the toolkit for understanding provided by the technique of deferents and epicycles, which quite clearly illustrates the virtuously circular connection between understanding and explanations that de Regt defends. With ingenuity and geometric knowledge (once again the subjective-pragmatic factors), Hipparchus was able to determine the mathematical and observational parameters needed to build an accurate solar model from Apollonius' insight.[28] The empirical success of this model motivated the quest for a comprehensive model for all celestial objects using deferents and epicycles. That is, the empirical success of the scientific explanation built using a toolkit of understanding started the process its canonization as a toolkit for UT and UP. A comprehensive model for the planets required further refinement of the tool, and this task was accomplished by Ptolemy. He developed the mathematical structure introduced by Hipparchus, significantly enhancing the scope of empirical adequacy of the model. Given the empirical success of Ptolemy's model, the toolkit of deferents and epicycles gets fully canonized as a standard of intelligibility.

[28] An interesting contextual difference is that the deferent-epicycle model is intelligible to us in a simpler and easier way because we have modern trigonometry, to which Hipparchus did not have access. His calibration of the solar model was much more tortuous than for us.

The development of medieval astronomy consists mostly in amendments of parameters in Ptolemy's model in order to cope with newly found phenomena—actually a mixture of real and false effects like trepidation and apogee precession—but always using the same geometric-kinematic toolkit of intelligibility. As we saw, in the 8th century, Al-Battani amended some empirical and geometric parameters in the model in order to make it fit better with observations, and Thabit Ibn Qurra introduced a theory of trepidations—in both cases the toolkit of intelligibility is still the method of deferents and epicycles, of course. We also saw that Ibn Al-Shatir in the 14th century introduced a model in which he showed that the equant can be dispensed with using a method of epicycles on epicycles. Besides, he also amended Ptolemy's model in order to cope with solar apogee precession. That is, Al-Shatir, further improved the empirical adequacy of the deferent-epicycles method, and he brought it back to coherency with circular-*uniform* motion.

Even Copernicus' heliocentric model can be understood in this way. Although the Polish astronomer explained away retrograde motion, the whole heliocentric model is still built on the method of deferents and epicycles—recall that the observed variable angular velocities of the planets are accounted for by epicycles, and that complex arrangement of epicycles on epicycles manages to avoid the introduction of the equant, retaining coherence with the principle of circular uniform motion. Something similar holds for Brahe's model. The Danish astronomer, exploding the inter-translatability between the Ptolemaic and the Copernican models, formulated yet another astronomical system—that captured the best of both worlds—using the deferent-epicycle method as his toolkit for model construction.

Summarizing, Eudoxus' method of concentric spheres was the invention of a toolkit of intelligibility, insofar as it made possible the construction of a model out of the law of circular uniform motion. Apollonius introduced a second set of tools of intelligibility for the same law, which led to the construction of several models by subsequent astronomers. The increasing empirical success of all the deferent-epicycles models from Hipparchus to Brahe is a clear example of the mutual feedback between understanding and explanation. The toolkit of intelligibility given by the method of deferents and epicycles made possible the construction of several explanatory models, the increasing empirical success of which led to its canonization as a source of understanding. Conversely, the refinement of the toolkit motivated by its fruitfulness in model construction led to the development of better explanatory models.

The relevance of Kepler's work for toolkits of understanding in the historical development of astronomy can be understood in a twofold way. First, he introduced yet another tool of understanding. Although the vicarious hypothesis resembles the previous models, it is certainly the outcome of a novel way to do astronomy. Despite the retention of circular motion, deferents and epicycles are fully abandoned. Besides, the choice of the real Sun as the anchor of the whole model was a revolutionary maneuver. Kepler's new method to build the vicarious model led to important advances in scientific intelligibility: the theory of latitudes could be greatly simplified, it allowed more precise determinations of planetary orbits (recall the improvement in the description of the terrestrial orbit), it allowed several methods to determine the inclination between planetary orbital planes and the ecliptic, and it allowed different ways to estimate the way in which the Sun, the center of the orbit and the equant were arranged. In short, Kepler's new astronomical method resulted in a better model, in the sense that the model scored higher in terms of intelligibility. The novelty of Kepler's approach can also be seen in that even though it retained circular motion, the underlying view of nature that it made scientifically intelligible was not the ancient Greek law, but his revolutionary conception of a mechanical universe. That is, Kepler, based on a sharp and insightful analysis of Copernicus' model, came up with a new dynamical-mechanical principle for the understanding of natural phenomena (UP), and he developed a new approach to make that principle scientifically intelligible through the vicarious model (UT).

On the other hand, despite all the improvement in intelligibility that the vicarious model allowed, he soon noticed that the model was false. Now, since all the models were geometrically inter-translatable, the falsity of the vicarious hypothesis was also a demonstration that all the previous models are equally false. Thus, Kepler's work signified the ruin of the metaphysical bases and geometric methods of ancient astronomy. Kepler showed that both the law of Greek motion and the toolkit of deferents and epicycles were dead ends. A most interesting aspect is that despite this, his new metaphysical-theoretical mechanical principle for the understanding of astronomical phenomena was not falsified by the empirical failure of the vicarious model. On the contrary, further elaboration on the false vicarious model led him to the formulation of the area law, a crucial landmark in the history of science. That is, although his first attempt to make the mechanical principle of understanding the phenomena scientifically intelligible led to a false model, the false model nevertheless led him to spectacular success, a success that in the long run led to the canonization of Kepler's way to do astronomy as a toolkit for scientific understanding.

3.2. Differing Standards of Scientific Understanding

We have seen how the deferent-epicycle method raised and fell as a canonical toolkit for understanding in model construction. Our case-study also illustrates another aspect that is central in de Regt's proposal. Recall that the criterion for intelligibility of theories states that a theory is intelligible to scientists *in a context*. That is, there are further pragmatic criteria, apart from usefulness for the construction of empirically adequate models, which determine the evaluation of a theory or model as a source of scientific understanding. Given the interplay between UT and UP, we saw that the canonization of a toolkit for model construction also leads to its canonization as a tool for understanding *the phenomena*. In our case-study, the evaluation of astronomical models in terms of the UP they offered clearly illustrates the relevance of pragmatic factors. Although all the models after Eudoxus and before Kepler were crafted using the toolkit of deferents and epicycles, the resulting models conveyed different forms of intelligibility *of the phenomena*, and there was significant divergence in the evaluation of each of those forms of intelligibility.

Although Ptolemy's model was universally accepted as the most accurate representation of the observed phenomena, there were important critical voices that considered the model unsatisfactory. The main targets of criticism were the introduction of several centers of circular motion, and the use of the equant. These features involved friction with principles of Aristotelian physics, and even with the aesthetic-metaphysical foundations of the Greek law of circular uniform motion. The teleological nature of circular motion of celestial bodies made essential reference to the center *of the universe*, but Ptolemy's model included several different centers for circular motion: what was special about those empty points that defined natural circular motion around them? On the other hand, the equant, as we saw, was in open conflict with the uniformity of motion. Based on these two problems, prominent medieval astronomers and philosophers did not accept Ptolemy's model as a source of intelligibility *of the phenomena*, at most, they took it as an instrument for prediction (see Goldstein 1980; Sabra 1984; Nutkiewicz 1978; Saliba 1991).

In the 11th century, Ibn al-Haytham, in his *Doubts Concerning Ptolemy*, stated that Ptolemy's model was plainly false, given its inclusion of the equant and its resulting friction with the Greek law of motion and with Aristotelian physics, and in his *Models of the Motions of Each of the Seven Planets* he attempted a model that got rid of the equant (see Rashed 2007). In the same spirit, in the 12th century Averroes expressed the following evaluation of the Ptolemaic system:

> For to assert the existence of an eccentric sphere or an epicyclic sphere is contrary to nature. As for the epicyclic sphere, this is not at all possible; for a body that moves in a circle must move about the center of the

> universe, not aside from it … It is similarly the case with the eccentric sphere proposed by Ptolemy […]. For nothing of the [true] science of astronomy exists in our time, the astronomy of our time being only in agreement with calculations and not with what exists. (Quoted in Çimen 2019, 141)

Also during the 12th century, Al-Bitruji, motivated by his dissatisfaction with Ptolemy's model given its friction with circular uniform motion, formulated a model of concentric spheres—similar to Eudoxus' (see Çimen 2019; Goldstein 1971). This model, though, was not able to match the empirical accuracy of Ptolemy's system. As we saw above, even as late as the 16th century, just before the introduction of Copernicus' model, Girolamo Fracastoro (in his *Homocentrica*, published in 1538) and Giovanni Batista Amici (in his *On the Motion of Celestial Bodies*, published in 1536) attempted to revive the system of concentric spheres, motivated by a strict observance of Aristotelian physics (see Dreyer 1953, 296-304). Finally, al-Shatir in the 14th century, also based on his dissatisfaction with the friction between Ptolemy's model and the principle of circular uniform motion, invented a method to get rid of the equant by a method of epicycles on epicycles (see Kennedy and Roberts 1959).

All of these criticisms to Ptolemy that motivated attempts to construct alternative models, quite clearly show that during medieval times there were diverging views regarding the value attributed to Ptolemy's model (and even to the method of deferents and epicycles itself, in the case of Al-Bitruji, Averroes, Fracastoro and Amici) as a source of understanding of celestial phenomena. Actually, the *Maragha School*—an Arabic school of astronomy founded around the Maragha observatory, of which al-Shatir was a member—was a whole scientific sub-community that challenged Ptolemy's system as a canon of understanding, based on the objections concerning its friction with uniform circular motion (see Saliba 1991). This clearly confirms that other factors apart from empirical adequacy—such as the commitment to uniform motion, or to strict Aristotelian physics—were crucial, *in some contexts*, for whether a certain theory or model is considered as conveying understanding of the phenomena or not. Actually, that these criticisms appeared in the heyday of scholasticism, whereas during Ptolemy's own time and the subsequent centuries there are no records of similar complaints, helps to explain that the criticisms respond to the contextual factor of adherence to Aristotelian views:

> The great conflict between Ptolemaic astronomy and Aristotelian cosmology—which continued right up until the sixteenth century—did not exist when originally Ptolemy wrote the *Almagest*. Ptolemy wrote his masterpiece 500 years after Aristotle, and the whole issue of the physical interpretation of his theory clearly was of secondary importance to him, and presumably to his contemporaries. However, to those rediscovering these works, the time between Aristotle and Ptolemy was not significant—they were both part of the 'ancient learning' that was being resurrected—and taken together they appeared full of contradictions. (2004, 100)

A similar pattern can be recognized in the rivalry between Copernicus and Ptolemy, and later between Copernicus and Brahe. A choice between the Copernican and Ptolemaic systems by, say, 1560, certainly depended on the different evaluations that different astronomers made of the conceptual advantages of one model over the other. That is, since the models were predictively equivalent (at least until the introduction of the telescope), which of the models ranked higher in terms of understanding of the phenomena clearly depended on the value attributed to their clusters of virtues and problems. As it was natural, for most astronomers of the time the price of an Earth in motion was too high—for there were no conceptual basis on which to make sense of it, and no serious contenders for Aristotelian terrestrial physics. The conceptual virtues of the Copernican model, though, were too important to be ignored, so the general attitude was to use the heliocentric model for calculations, but still adhering to Ptolemy as the true model. An illustrative example of this stance is given by the astronomer Thomas Blandeville, who in 1594 wrote "Copernicus […] affirmeth that the earth turneth about and that the Sun standeth still in the midst of the heavens, *by help of which false assumption* he hath made truer demonstrations of the motions and revolutions of the celestial spheres, than ever were made before" (quoted in Kuhn 1957, 186; my emphasis).

However, there were astronomers that endorsed the heliocentric model. Soon after the publication of *On the Revolutions*, in the trade between the natural explanation of retrograde motion and the trigonometric method to determine planetary distances in Copernicus' model, on the one hand, and the problems associated to an Earth in motion, on the other, some prominent astronomers ventured to value the former features highly enough as to tolerate the latter. Georg Joachim Rheticus, an Austrian pupil of Copernicus who became a professor in Leipzig and in Wittenberg, was an early advocate of a literal interpretation of the heliocentric system. Rheticus was a driving force behind the publication of *On the Revolutions*, and in 1540 he published an introduction to Copernicus' model entitled *First Account of Copernicus Book on the Revolutions*, in which he defended the model both in mathematical and physical terms (see Danielson 2006). Michael Mästlin, a German astronomer that worked in Tubingen, was another important early Copernican. He taught and defended the model in Tubingen, and it was under the mentoring of Mästlin that Kepler became a Copernican too (see Methuen 1996). In England, in 1576 Thomas Digges wrote a review of the heliocentric model in which he defended it as a true model.

This of course illustrates that there is synchronic variation of criteria of understanding. The majority of astronomers of the time, for good reasons, included a geocentrism constraint in the assumed standard of intelligibility of celestial phenomena. The dominance of the commitment to static Earth physics can also be seen in that after Brahe's work most of the supporters of Ptolemy converted to the Tychonic hybrid model—which, as we saw, was able to grasp most of the conceptual appeal of Copernicus' model, but in a geocentric setup. For scientists like Rheticus, Mästlin, Digges, Kepler and Galileo, though, the conceptual appeal of the Copernican model ranked higher than coherence with then accepted terrestrial physics. These astronomers considered that true understanding of the phenomena was conveyed by the Copernican model, even to the extent of tolerating the absence of a dynamical account of terrestrial phenomena compatible with a moving Earth. Actually, Kepler and Galileo were crucial in taking important steps towards a new physics of an Earth in motion. This illustrates that the context-dependency of understanding, with the resulting synchronic variation of standards of intelligibility of the phenomena, can be an engine for scientific progress.

3.3. Understanding and Metaphysics

An interesting subtlety in de Regt's account of scientific understanding is given by its connection with metaphysics. He introduces the concept of *metaphysical intelligibility*: "a theory is metaphysically intelligible if it harmonizes with extant, or preferred, metaphysics" (2017, 160). This notion differs from the scientific intelligibility involved in UT. However, he argues that metaphysical and scientific intelligibility interact, for the former can provide tools to render a theory scientifically intelligible in the sense of UT.

Our case-study also provides support for this view. The ancient Greek law of circular uniform motion is actually rooted in metaphysics. As we saw, Plato argued that this type of motion essentially corresponds to celestial objects because the demiurge decided to introduce time in nature, and circular uniform motion is appropriate for that task. Furthermore, Aristotle, based on aesthetic properties of circular motion and teleological considerations, concluded that this must be the form of natural motion corresponding to the ethereal element. That is, the two greatest ancient Greek philosophers attributed circular uniform motion to celestial bodies based on metaphysical views, and these views led to the construction of astronomical models. A clear indication of this is given by Simplicius comment that Eudoxus' model was developed as a response to Plato's challenge of representing celestial motion by a geometric system coherent with the described metaphysical views. Thus, the metaphysics of nature endorsed by Plato and Aristotle prompted the invention of toolkits of scientific intelligibility: the method of concentric spheres and the method of deferents and epicycles.

An even clearer example of the interplay between metaphysical and scientific intelligibility is given by Kepler's vicarious model. Recall that already in the *Secret of the Universe*, published in 1597, Kepler was already convinced that the Sun was responsible for the choreography of the planets by the action of a force. That is, he abandoned the teleological picture associated to the doctrine of circular uniform motion, and embraced a proto-mechanical metaphysical picture of the universe. In 1609, in the *New Astronomy*, Kepler introduced the vicarious model by amending Copernicus' model with his revolutionary metaphysics as a heuristic principle. As we saw, he fully dismissed the method of deferents and epicicyles, and assigned a circular orbit to each planet. All orbital planes intersect in the Sun, and the center of each orbit lies between the Sun and the equant, which implied that the planet's speed is fastest at perihelion and slowest at aphelion—in coherence with the view that the strength of the solar force diminishes with distance. We also saw that this arrangement was a huge improvement in model construction. In the vicarious model, the latitude of the planets is simply a function of the inclination of the orbital planes with respect to the ecliptic, whereas in the previous models the latitudinal theory was rather contrived and inexact. Furthermore, Kepler's model allowed geometric methods to determine such inclination, to determine the distances between the Sun, the orbit center, and the equant, and to determine (and correct) the circle of the Earth's orbit. That is, Kepler's revolutionary conviction to a principle of metaphysical intelligibility—a mechanical universe—led to an improvement in scientific intelligibility in astronomical modelling.

Now, the failure of Kepler's vicarious model, which as we saw was built on the basis of a proto-mechanical metaphysics as heuristic principle, led in turn to further scientific progress in theory and model construction. The unavoidable empirical inadequacy of 8' in the vicarious model—and consequently in all the extant models of the time—signified the end of the principles of ancient astronomy. Kepler fully abandoned circular motion, but he remained fully committed to his proto-mechanic metaphysics. Exploiting the fact the orbital motion is fastest at perihelion and slowest and aphelion, in connection with the thesis of a Solar force proportional to distance that he took as the foundation of such fact, Kepler obtained his second law (and also the first one, see Barbour 2001, Ch. 6; Torretti 2007, Ch. 4).

The specific details of Kepler's proto-mechanical picture were wrong, of course. The nature of the solar force he conceived was analogous to fan blades: the Sun emits a force similar to light rays, and given a solar rotation that Kepler postulated, the rays of force drag the planets along their circular orbits. Besides, Kepler conceived the force as inversely proportional to the distance rather than to the squared distance, and as producing a velocity rather than an acceleration. His efforts to find a mathematical expression for the solar force he envisioned were thus fruitless. However, Kepler's proto-mechanical metaphysical picture was crucial in the construction of the vicarious model, and in the formulation of his first two laws. Furthermore, the proto-mechanical picture that Kepler introduced had a crucial influence on Descartes' systematic exposition of the mechanical philosophy—which in turn had a crucial relevance in the constitution of modern physics—and on Newton's formulation of the universal law of gravitation.

Despite its success as an engine of scientific understanding, Kepler's proto-mechanical metaphysics was not immediately accepted. A clear example of this is given by Ismael Boulliau's *Astronomia Philolaica*, one of the most important treatises in astronomy between Kepler and Newton, published in 1645. There Boulliau openly acknowledged that Kepler's laws allow a geometrical model that is empirically superior to all the preceding proposals. However, he rejected Kepler's underlying physics, and he introduced a curious model in which the elliptical orbits result from combinations of circular motions. Furthermore, Boulliau proposed that the ultimate foundation of planetary motion was not an external force, but an internal principle given by the essence of celestial objects, pretty much in the spirit of ancient astronomy and Greek teleological metaphysics (for an analysis of Boulliau's work, see Wilson 1970). Although Descartes' (1979) vortex theory in *The World*, published in 1664, became an influential attempt of a theory of the solar system based on mechanistic principles, the adoption of mechanistic metaphysics in scientific practice, and the full abandonment of teleology was gradual, and culminated with Newton's work.

All of this offers further support for de Regt's account of understanding. The diverging attitudes of astronomers of the time towards the mechanistic framework as a source of metaphysical intelligibility illustrates that this form of intelligibility is also pragmatic and context dependent, just like scientific intelligibility. Besides, the rise and fall of teleological and circular motion metaphysics, and its replacement with the mechanical picture, illustrates the interplay between metaphysical and scientific intelligibility. The success of a certain canon of metaphysical intelligibility as a source of scientific intelligibility can lead to the canonization of the metaphysical picture, but this canonical status is conditional on the coherence of the metaphysical picture with successful science. The teleological circular motion metaphysical framework was dominant for about two millennia, until the mechanistic picture showed to be much more fruitful as a metaphysical background for the construction of scientific knowledge.

3.4. Understanding from False Models

There is one last very interesting sense in which our narrative of the history of astronomy from Eudoxus to Kepler gives support for the account of understanding defended by de Regt: scientific understanding of natural phenomena can be obtained from false models. Let us recall de Regt's criterion for understanding a phenomenon (CUP): "a phenomenon P is understood scientifically if and only if there is an explanation of P that is based on an intelligible theory T and conforms to the basic epistemic values of empirical adequacy and internal consistency" (2017, 92). So far we have paid close attention to the first condition for UP, namely, an intelligible theory from which an explanation for P can be built. The second condition of internal consistency and empirical adequacy of T, at face value at least, seems to amount to a condition of veridicality, i.e., that the explanatory model built out of T must be (approximately) true in order to convey scientific understanding.

However, de Regt & Gijsbers (2017) reformulate the second condition in such a way that it is clearly coherent with the view that scientific understanding can be obtained from explanatory models that are (even wildly) false. Instead of in terms of empirical adequacy and consistency, in the updated formulation of the CUP de Regt & Gijsbers introduce the second condition in terms of the wider concept of *reliable success*. That is, P is scientifically understood iff there is an explanatory model of P based on an intelligible T, and such that the model is reliably successful.

The concept of reliable success is defined in terms of three tasks: making correct predictions, guiding practical applications, and developing better science. The goal of making correct predictions captures the idea of empirical adequacy. Now, reliable success does not require that this goal is attained by means of strict veridicality. That is, explanatory models that, although patently false, have a certain degree of predictive success, can still be reliably successful. In the definition of reliable success none of the three tasks has a privileged status over the other two. Most naturally, scientists will always prefer explanatory models that rank high in all three tasks, but in the absence of such models, they can settle for less. Thus, false theories or models that in a certain range of phenomena predict reasonably well, and which have important practical applications, or that can open paths to new and better science, can comply with the second constraint for providing UP.

Just like scientific intelligibility, reliable success is a pragmatic notion. The assessment of how a certain model ranks in all three tasks is clearly sensitive to the context of evaluation. For example, the degree of success in the first task depends on the realm of phenomena considered—this is why I stated that false models that predict *reasonable well* can still be reliably successful: which degree of predictive success counts as reasonably good is a pragmatic issue. For example, Newtonian mechanics is empirically successful in terrestrial phenomena: it works well for mid-sized objects and velocities much lower than c, but it fails in other contexts. Accordingly, de Regt & Gijsbers (2017, 57) state that a theory T is reliably successful *for a scientist S iff S can use a theory in her scientific work* in such a way that T performs well in at least one of the three mentioned tasks. The reference to S and to her scientific work captures the context-dependent element in judgments about reliable success.

Kepler's vicarious hypothesis constitutes a clear vindication of the thesis that scientific understanding of phenomena can be obtained from a wildly false model. As we saw, the heuristic principle in Kepler's construction of the vicarious model was his proto-mechanical framework for the physics of the solar system. Now, as soon as he attempted the task of determining the parameters in the model, he noticed that the ratio between the distance from the center of the orbit to the equant and the distance from the center of the orbit to the Sun yielded different values depending on whether it was calculated from the latitude or the longitude of Mars, a conflict which in turn led to an unavoidable empirical error of 8'. That is, as soon as he formulated it, Kepler knew that the model was false.

However, he was explicit about the crucial scientific value that the false model had. The title of chapter 21 of the *New Astronomy* is entitled *Why, and to what extent, may a false hypothesis yield the truth?* There Kepler stated that despite its falsity, the vicarious model does capture some true aspects in the kinematic configuration of the solar system. For example, the model does predict the right longitude at some points in the Martian orbital trajectory. In Kepler's own words:

> There are, however, occasions upon which a false hypothesis can simulate truth, within the limits of observational precision, with respect to the longitude. (Kepler 1992, 294)

> It is at least now clear to what extent and in what manner the truth may follow from false principles: whatever is false in these hypotheses is peculiar to them and can be absent, while whatever endows truth with necessity is in general aspect wholly true and nothing else.
> Further, as these false principles are fitted only to certain positions throughout the whole circle, it follows that they will not be entirely correct outside those positions, except to the extent […] that the difference can no longer be appraised by the acuteness of the senses. (1992, 298)

In these passages, Kepler is explicit in that the model, despite being false, has a degree of empirical adequacy that is enough to assign it a scientific value. Now, such value does not reside only in the degree of predictive success. If that were the case, the same value could be attributed to the three rival models (Ptolemy's, Copernicus', and Brahe's), given their geometric intertranslability.

The scientific value that Kepler assigned to his vicarious model, which we can read in terms of de Regt & Gijsbers' characterization of UP from false models, relied in its scientific fruitfulness. Unlike its contenders, the vicarious model was able to open an avenue to better science. As we saw above, Kepler formulated his crucial second law in the context of the vicarious model: in Kepler's own formulation, the law does not refer to the elliptic character of the orbits. Actually, the first law of the elliptic orbits was found by Kepler from the empirical failure of the vicarious model and the orbital predictions from the area law. As it is clear from our exposition of the different astronomical models of circular motion, this trail to better science could only be blazed by the vicarious model. Thus, Kepler's vicarious hypothesis yields an UP that cannot be obtained from the preceding models. Despite the fact that they are all equally false, the area law could be formulated from the vicarious hypothesis, but not from the Ptolemaic, the Copernican or the Tychonic model.

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